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THE UNDERGROUND GEOPHYSICAL STATIONS AND THE HORIZONTAL PENDULUMS OF THE OBSERVATOIRE ROYAL DE BELGIQUE

J. Verbaandert and P. Melchior

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OBSERVATOIRE ROYAL DE BELGIQUE

[ROYAL OBSERVATORY OF BELGIUM]

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LES STATIONS GEOPHYSIQUES SOUTERRAINES ET LES PENDULES HORIZONTAUX DE L'OBSERVATOIRE ROYAL DE BELGIQUE

[THE UNDERGROUND GEOPHYSICAL STATIONS AND THE HORIZONTAL PENDULUMS OF THE OBSERVATOIRE ROYAL DE BELGIQUE]

by

J. VERBAANDERT AND P. MELCHIOR

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THE UNDERGROUND GEOPHYSICAL STATIONS AND THE HORIZONTAL PENDULUM OF THE ROYAL BELGIAN OBSERVATORY

by J. Verbaandert and P. Melchoir

INTRODUCTION

As a consequence of the continually increasing precision of scientific measurements, the influence of the terrestrial tide is manifested in the observation of many phenomena which are at first sight totally unrelated. Many instruments anchored to the crust of the earth undergo disturbances by the deformation of the earth and, as soon as their precision becomes sufficient, the measures derived from them are systematically altered and must be appropriately corrected before lending themselves to valid interpretations.

We must therefore understand the phenomenon if we desire to calculate the necessary corrections. Obviously the "utilitarian" aspect is not the essential object of research and it will not be difficult to understand that the geophysicist places the greatest importance on the experimental data concerning the terrestrial tides because they afford him a new means of investigating the physical characteristics of the earth.

The possible deformations of the ground must today be known with an increasingly greater precision. The best proof of this is found in the difficulties encountered in the construction of the proton synchroton of CERN at Geneva where very accurate geodetic measurements had to be carried out by a metrological team. Leveling had to be carried out with a very high degree of precision because the protons perform some 500,000 revolutions in the accelerator before being precipitated on the target. The slightest divergence of the magnets from the ideal circle causes them to rapidly deviate from their trajectory and become lost.

Observations have demonstrated an appreciable effect of the terrestrial tides over a long period (14 days) and special arrangements had to be devised by the engineers in order to eliminate their disturbing influence.

Brief Description of the Phenomenon

Any point on the surface of the earth is subject to two forces which are the force of gravity due to the attraction of the total mass of the earth and the centrifugal force due to the rotation of the earth. The resultant of these two forces is a vector oriented toward the center of the earth whose length represents the intensity of gravity at the respective point and whose orientation defines the direction of the vertical of the locus. These last two elements cannot be considered as strictly constant because the moon and the sun exercise attraction on the respective point and this attraction varies in time with the position of these celestial bodies. This is the phenomenon giving rise to the maritime tides which manifest the fact that the fluid surface of the oceans adapts itself at any instant to the surface of the level perpendicular to the "disturbed" verticals. The specific circumstances of the different oceans such as their fluidity and dimensions producing resonance with the disturbing force (although in a highly diversified manner), make observation simple and direct but render the theory of their manifestations very nearly indefinable.

Located on an absolutely rigid base and equipped with high-precision instruments, we would be able to directly observe the periodic variations from the vertical (they can reach an amplitude of 0.02") and the periodic variations of the intensity of gravity (which attain about 0.2 milligal or 2×10^{-7} g) due to the variable attraction of the sun and the moon. Under these conditions, we would be able to verify the coefficients provided by the astronomical theories (lunar and solar tables).

However, this is pure theory because we cannot conceive that the crust of the earth or, strictly speaking, the whole of the terrestrial globe is infinitely rigid and are forced to admit from the start that the earth possesses a certain elasticity and viscosity. The mathematical formula of the relation elasticity- viscosity belongs to the field of terrestrial rheology which is as yet not known and for which only approximate solutions have been suggested. Consequently, the earth becomes deformed under the influence of the lunar-solar potential of attraction and the amplitude as well as perhaps the phase of the deviations from the vertical and the variations of the intensity of gravity vary in relation to the theoretical values furnished by the science of celestial mechanics.

In order to more fully define these changes, let us keep in mind that the deviations from the vertical are recorded by an instrument (the horizontal pendulum) rigidly anchored to the crust. The latter becomes deformed in the direction of the force of attraction to which it is subjected and this results in a certain compensation in the amplitude of the effects recorded by the instrument. By contrast, the gravimeter recording a minimum of the intensity of gravity at the instant where the moon is in the zenith of the locus, is lifted up with the crust due to the force of the tide. Consequently it moves away from the center of the earth and the reduction of gravity is further increased. The change in amplitude consequently is an action of amplification for this instrument.

Moreover, we must not forget that these deformations of the earth change not only the phenomena existing in the case of a rigid globe but also give rise to tensions and spatial dilations which would be non-existent in a rigid globe.

The development of the lunar (or solar) potential of attraction at a point of the earth (defined by its latitude and longitude) made it possible for Laplace to demonstrate three types of tides (this is valid both for terrestrial and maritime tides) which correspond to the three types of spherical harmonic functions of the surface. They are represented in Fig. 1 and their characteristics are the following:

A) The first of these functions has as nodal lines (lines where the function is canceled out) only the meridians located at 45° on either side of the meridian in which the celestial body is located; these lines divide the sphere into four sectors where the function is alternately positive or negative; the regions where the potential W is positive are those of the high tides and the negative regions are those of the low tides. The function is called <u>sectorial function</u>, the period of the tides corresponding to this function is semidiurnal and their amplitude is maximum at the equator when the declination of the respective heavenly body is zero. Both are zero at the poles. Let us remember that the variations in the distribution of mass at the surface of the earth consequent on the sectorial distribution do not modify either the position of the pole of inertia or the large moment of inertia C (on which the speed of rotation of the earth depends).

B) The second function has as nodal line a meridian (at 90° from the meridian of the respective heavenly body) and a parallel, i.e., the equator. This is a <u>tesseral function</u>; the regions into which it divides the sphere change sign with the declination of the heavenly body. The period of the corresponding tides is diurnal and the amplitude is maximum at latitudes 45° N and 45° S and also when the declination of the heavenly body is maximum; it is always zero at the equator and at the poles. The variations

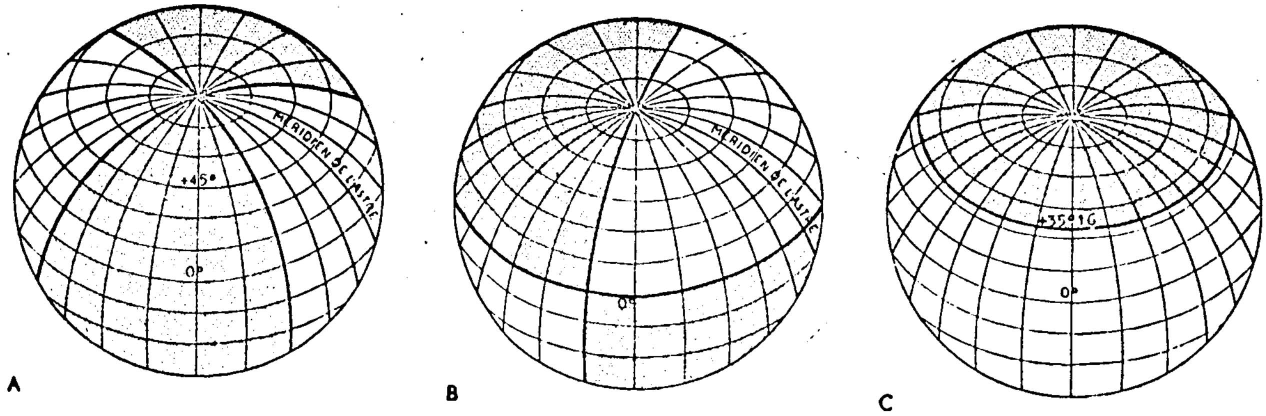


Fig. 1 - Geometrical representations of the three kinds of tides.

in the distribution of mass at the surface of the earth subject to tesseral distribution vary the position of the pole of inertia but not of the large moment of inertia C.

C) The third function depends only on latitude and is a zonal function; its nodal lines are parallels of + and - 35°16". Since it is moreover only a function of the square of the sine of the declination of the heavenly body, its period will be 14 days in the case of the moon and 6 months in the case of the sun. The variations of distribution of mass at the surface of the earth corresponding to the zonal distribution do not displace the pole of inertia but change the large moment of inertia C. We must therefore anticipate variations in the speed of rotation of the earth having the above periods. They can actually be detected by Time Services equipped with ultra-precise instruments. The surface level will be lowered by 28 cm at the pole and raised by 14 cm at the equator: the effect of this permanent tide is to very slightly increase the constant flattening of the earth (the difference between the large axis and the small axis of the terrestrial ellipsoid is 21.37 km).

Each type of tide can be decomposed into a very large number of terms depending on the different characteristics of the orbits of the moon and the earth. We shall further discuss this at the end of this report by presenting some numerical findings obtained with horizontal pendulums which are the main objective of the report.

Terrestrial Tides and Astronomy

We can easily imagine the influence of the terrestrial tides on basic astronomical observations.

The astronomic latitude of an observatory is the angle formed by the vertical with the celestial equator which consequently undergoes periodic variations corresponding to the deviations of the vertical. T. Shida was the first to demonstrate them in observations at the Latitude Service. The longitudes are also disturbed but this has not yet been investigated.

The operation of astronomical pendulum clocks is a function of the instantaneous value of gravity and consequently is subject to tidal variations which were investigated for the first time in 1930 by Brown and Brouwer. These variations can presently be observed by continuously recording on a drum chronograph both the oscillations of the pendulum and of the quartz clocks.

Long-period tides corresponding to a zonal distribution of the deformations cause variations of the largest moment of inertia of the earth and consequently variations of the speed of rotation of the latter. Anticipated by H. Jeffreys in 1928, their existence was confirmed experimentally by Wm. Markowitz (their amplitude is 0.001 second of time).

The dissipation of energy implied by the deformations of the terrestrial tides furnishes at least a partial explanation of the progressive deceleration of the rotation of the earth.

Terrestrial Tides and Geodetics

Here again the connection is obvious and we see immediately that high-precision surveying would disclose systematically periodic effects. The hydrostatic surveys of Norlund have proved that the phenomenon is perceptible even though it has not yet been eliminated from operations on the ground. However, correction formulas for the tidal effect are already being applied by certain geodeticists.

However, geodetics is not only interested in the periodic deformation of surface level and of the geoid in particular. It is also considerably interested in the small variations of the intensity of gravity which must be taken into account in high-precision gravimetric surveys.

Terrestrial Tides, Hydrology and Volcanology

We have already drawn attention to the fact that the deformations of the earth are accompanied by spatial stresses and dilations. The latter are manifested in the form of alternate compression and dilation or of decrease and increase of volume in relation to the radial deformation by the tides: at the instant of high tide where the crust rises, dilation occurs and compression takes place at the instant of low tide. It follows that any underground water table even at low depth (100 m and more) of the crust, whether in an actual cavity or in innumerable fissures like a sponge, undergoes these effects. They are manifested in the water level of a well which allows us to observe the state of the water table. It is obvious that, at low tide and at the instants of compression of the crust, the water is forced upward in the well whereas it will drop at the instant of dilation, i.e., during high tide. The phenomenon is consequently reversible and the water table plays the role of a pressure gauge.

All this is equally applicable to petroleum deposits, volcanic lava and all fluid, viscous, liquid or gaseous matter enclosed in the crust.

Terrestrial Tides and Internal Terrestrial Physics

Comparison of the experimental data with the theoretical values furnished by celestial mechanics allows us accurately to calculate the extent of the deformation and eventually the retardation (or advance) of this deformation in relation to the disturbing force. Such data can be established for each of the principal waves of the tide.

On the other hand, seismology furnishes relatively precise indications on the surfaces of discontinuity in the interior of the earth and on the most likely physical characteristics of each zone so defined. The deformations of the whole of the globe such as the terrestrial tides are a function of the radial distribution of the elastic densities and parameters (eventually of viscosity).

A fundamental investigation would therefore consist in constructing scale models of the interior of the earth in accordance with the data furnished by seismology so far. We can then solve the differential equations for deformation (equations of the sixth order) by means of an electronic computer and select, among the many solutions valid from the seismological point of view, those which agree with the observations furnished by the terrestrial tides. We would thus advance in our knowledge of the interior of the earth. The theoretical work has already been carried out but it became immediately apparent that the experimental data lacked precision and that it was absolutely necessary to make greater progress in this regard. The instrument which will be described in this report constitutes the result of a considerable effort in this direction and has already made it possible to substantially increase the precision of measurement of the deviations from the vertical.

Another question may be raised which is even more difficult to solve: Is there a systematic difference between the deformations in accordance with the period of the disturbing forces? In other words, are the diurnal periodic deformations affected differently by the elasticity of the globe than the semi-diurnal deformations? If this were so, this would indicate the fluidity of the terrestrial core but this has not yet been detected so that the question remains open.

For all information concerning the theory of the terrestrial tides, the organization of the observations throughout the world and the results obtained with various types of instruments (gravimeters, extensometers, horizontal pendulums), the reader is referred to a detailed publication(*).

We shall restrict our report here simply to the investigation of the <u>deviations from the vertical</u> carried out by means of instruments designed and constructed at the Royal Belgian Observatory.

Obviously, the investigation of the variations of intensity of gravity is also currently being pursued at the Royal Belgian Observatory but with German instruments (Askania). The findings have already been reported elsewhere(**).

(*) P. MELCHIOR, Earth Tides (Pergamon Press), being printed.
(**) P. MELCHIOR, Résultats de seize mois d'enregistrement de la marée à l'Observatoire Royal de Belgique (Uccle) a l'aide du gravimètre Askania n° 145 (1958-1959). Annales Obs. Royal de Belgique, VIII, 4, 135 pp., 1960.

Chapter I

CONSTRUCTION OF THE HORIZONTAL PENDULUM

The "horizontal pendulum" invented by Hengler and the first pendulum of Zöllner about 1830

With the discovery by Newton of the "law of universal gravitation" in the seventeenth century and its application to the interpretation of the phenomenon of the tides, it became evident that the lunar-solar attraction might impart considerable motion to moving bodies on the surface of the earth. A number of researchers certainly thought of attempting to measure the influence of such attraction on the equilibrium of a freely suspended vertical pendulum. Although such an experiment was even at that time easy to realize, we had to wait until the nineteenth century before the first communication on this subject was published.

In 1828, Gruithuisen at Munich published a report which aroused no particular interest at the time and in which he suggested suspending, in the shaft of a mine, a vertical pendulum with a length of 150 to 1,500 feet for the purpose of measuring possible deviations from the vertical in relation to the surrounding solid rock. The author also pointed out in his report that he had published in 1817 a note in which he described the experiments made by him with a pendulum 10 feet long in order to demonstrate the deviations caused by changes in the apparent direction of gravity and due, among others, to the variable attraction of the sun and the moon.

Before carrying out these experiments, Gruithuisen probably had not been able to calculate the theoretical deviations of his pendulum through lunar-solar attraction because he would have become immediately aware of the uselessness of his efforts in view of the fact that the combined attraction of the sun and of the moon are barely capable of imparting an angular displacement of 0.02" which would displace the end of the pendulum by no more than an amount slightly inferior to 1 micron and practically not measurable with the means available to him.

The great merit of Gruithuisen consisted in having been a pathfinder because we owe to one of his students, Lorenz Hengler, (born at Reichenhofen in 1806 and studying mathematics and astronomy at Munich from 1830 to 1831), the discovery of a device making it possible to multiply by an extremely large factor the micro-displacements of the end of a pendulum in such manner as to make them appropriate for experimentation. By conceiving of an ingenious bifilar suspension, Lorenz Hengler invented the horizontal pendulum (Fig. 2) to which he gave the name of "astronomical pendulum scale".

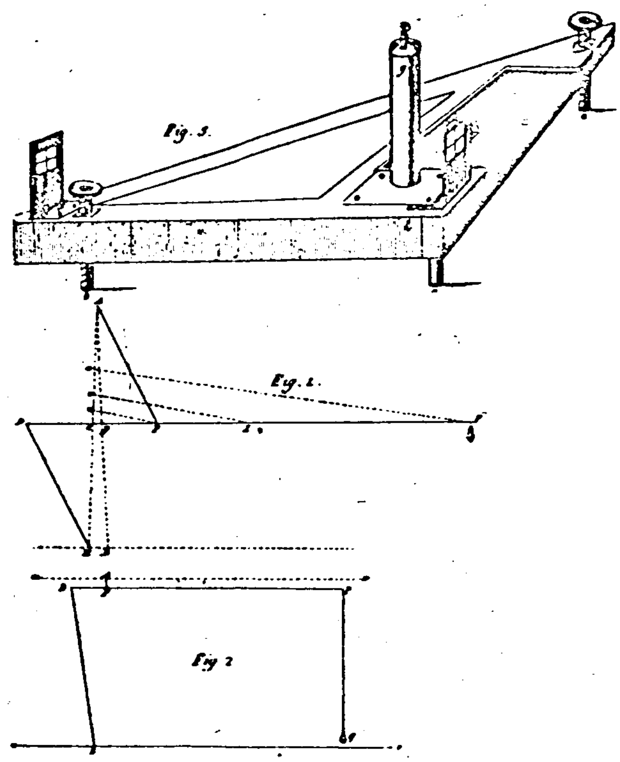


Fig. 2 - Reproduced from "Polytechnisches Journal von Dingler 1832". Fig. 1 corresponds to Hengler's diagram of his bifilar suspension; Fig. 2 is an attempt by Hengler to demonstrate the rotation of the earth by a laboratory experiment based on his bifilar suspension; Fig. 5 is a diagram of the horizontal pendulum on set-screws which he actually built.

With this instrument, he carried out numerous observations in order to demonstrate the effect of lunar-solar attraction. The movements of the end of the pendulum were observed by means of a microscope and the author stated that he was able to show a non-negligible real effect whose statistical averages of the measurements were in agreement with theoretical expectations.

If this remarkably gifted author did not arrive at absolutely positive results, this was due to the fact that he was ahead of his time in regard to his experimentation and did not yet possess the means developed subsequently such as photographic recording and the rotating-mirror method of Poggendorf which brought about the success of later observations.

Around 1872, Zöllner profited from the development of the above techniques and constructed a horizontal pendulum on truly scientific bases and built in fact according to the early concept of Hengler. Since then, bifilar suspension has had the name of "Zöllner suspension". The Zöllner pendulum made it possible to carry out the first measurements of real scientific value concerning both earthquakes as well as the slow movement of deformation of the ground.

Schematically, the Zöllner pendulum can be described as follows:

A very rigid upright P is mounted on a base plate with 3 set screws V_1, V_2, V_3 with which it can be leveled (Fig. 3). At A and B, points practically located on the same vertical, two thin metallic wires are attached which terminate in a horizontal metal arm at C and D. This produces an imaginary axis of rotation for the horizontal arm which passes through A and B and can be brought to coincide almost perfectly with the vertical by adjusting the set screws.

In order to make adjustment easier in the modern pendulums, the set screws of the base are preferably placed at the summits of a triangle with two equal sides.

The set screws V_1 and V_2 at the summits constitute the acute angles of the rectangular triangle and are called respectively "sensitivity screw" and "drift screw" of the horizontal pendulum for the following reasons.

By acting on V_1, we modify the inclination of the pendulum in the direction of the moving arm which thus rotates around a straight line passing through the base points of the two screws V_2 and V_3. In this case, if the above base points are approximately on the same horizontal, the moving arm should not undergo any lateral deviation except a possible reversal of 180° if the imaginary axis of rotation should be displaced beyond the vertical position.

On the other hand, by acting on the drift screw V_2, the pendulum is rotated around an approximately horizontal straight line passing through the base points of the screws V_1 and V_2 and the moving arm is subjected to a large angular deviation.

The foregoing considerations show that, with a single pendulum, it is possible to demonstrate only the components of deviation of the vertical which are manifested in a plane approximately perpendicular to the moving arm. In order to measure these deviations completely, we must therefore use two pendulums the arms of which have positions perpendicular to each other.

The mirror M attached to the pendulum arm makes it possible, by applying the Poggendorf method, to record photographically and continuously even the smallest movements of the pendulum. This then is the way the device functions (Fig. 5): A converging lens L with a focal length of several meters is placed in front of M; in the focus of L is located a small electric lamp e and the rectilinear filament of the latter is placed vertically. The luminous energy of the filament is reflected by M after first traversing L. Having been reflected, the light passes a second time through L and forms at I (behind a very narrow horizontal slot f), the punctiform image of the filament on a drum carrying a photographic film. The drum rotates uniformly and makes one complete

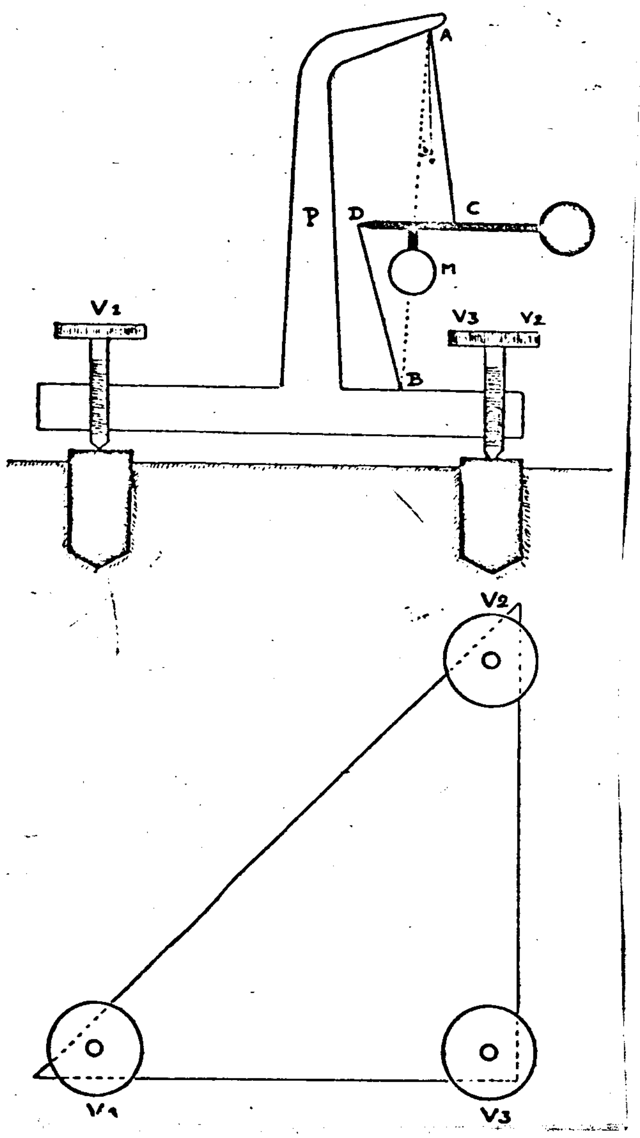


Fig. 3 - Diagram of the Zöllner suspension.

revolution per week. Consequently, the slightest rotation of the mirror deflects the reflected light beam by a double angle and the image I scans the photographic film in such manner that there are recorded continuously on the latter, with the rotation of the drum and in the form of a wave line as a function of time, the deviations of the mirror and the pendulum arm to which it is rigidly attached. Every 60 minutes, a small auxiliary lamp activated

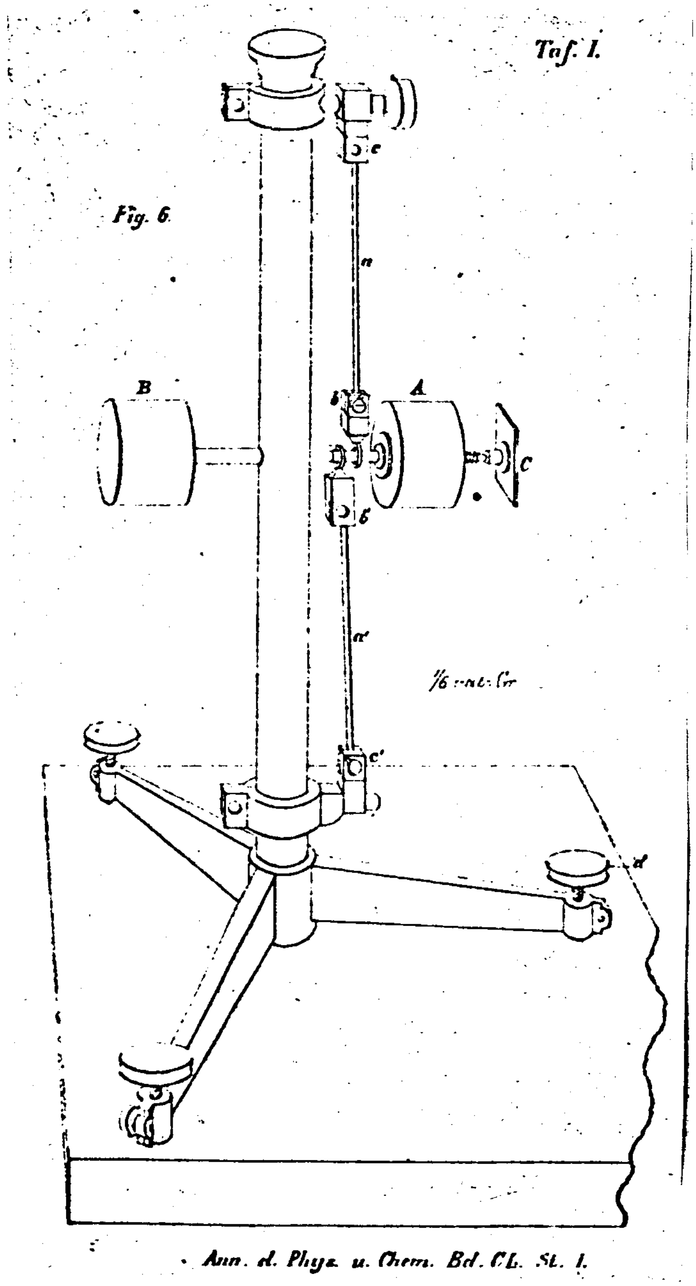


Fig. 4 - Reproduced from "Annalen der Physik und Chemie", vol. 150, 1873, and representing the pendulum built by Zöllner.

by a precision clock throws light on the slot for a few seconds in such manner that the time is recorded on the photographic film by parallel and equidistant lines.

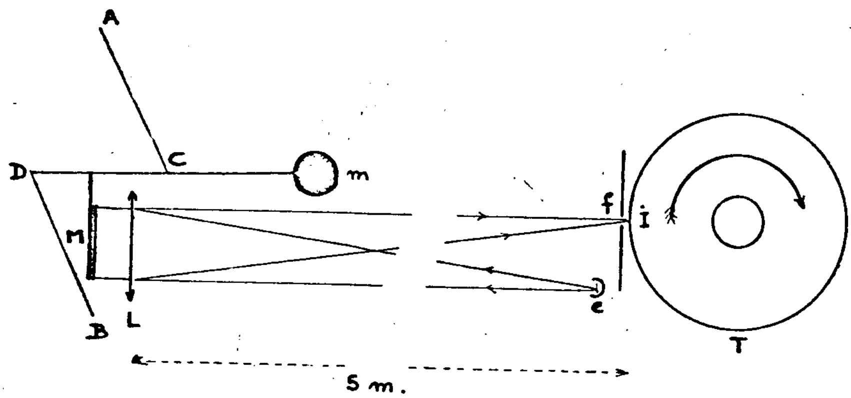


Fig. 5 - Diagram of recording by the Poggendorf method. M = plane mirror supported by the pendulum arm DCm; L = convergent lens with 5 m focus; e = electric lamp with vertical rectilinear filament; f = horizontal slot, 30 cm long and 0.1 mm wide; I = punctiform image of the lamp filament e projected after reflection from M on a point of the photographic paper wound on the drum T with the latter making one complete rotation per week.

The "Mach pendulum" as an instructive model of the horizontal pendulum

In order to make the characteristics of the horizontal pendulum clearer, we can turn to a didactic instrument found in many laboratories of applied mechanics and admirably illustrating the function of the former. This is the so-called Mach pendulum, named after its inventor, the Austrian theoretical physicist, made famous by his analyses of the principles of mechanics and also considered one of the forerunners of relativity.

Specifically designed to experimentally demonstrate the laws of oscillation of a pendulum, the instrument consists of a standard pendulum suspended within a frame as shown in Fig. 6. The frame can be inclined as desired in such fashion that the axis of rotation of the pendulum can move unhindered from the horizontal to the vertical position.

In constructing this pendulum for the purpose of instruction, Mach intended to demonstrate experimentally the law which links the period of oscillation T of the pendulum to the value g of the intensity of gravity. The

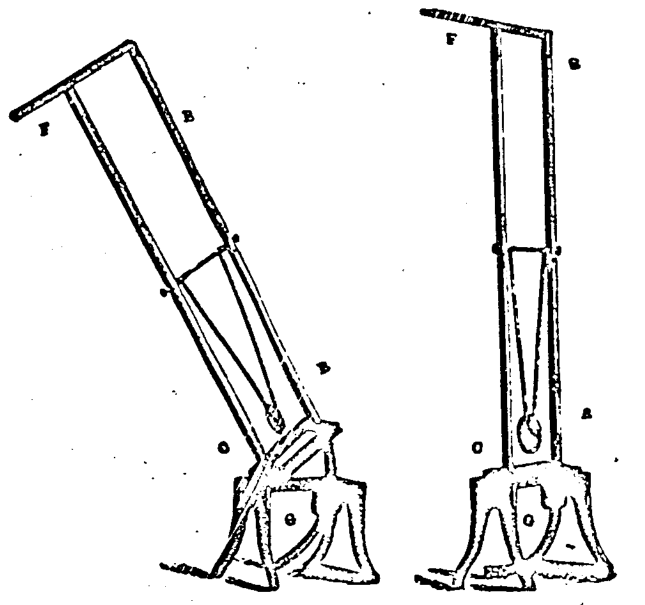


Fig. 6 - Reproduced from "La Mécanique" by Ernst Mach, 1908.
The graduated sector G allows measurement of the inclination i of the axis of rotation.

experiment is difficult because the value of g is not easily obtained. In order to obviate this difficulty, Mach employed a trick by apparently diminishing the value of gravity by inclining the axis of rotation of the pendulum. The frame is provided with a graduated quadrant which makes it possible at any instant to measure the angle between the axis AA' of rotation of the pendulum and the vertical plane. For any given inclination i, the component g sin i of gravity which acts in the direction of the line of the greatest slope of the plane of oscillation in such manner that we can write

$$T_i = 2\pi \sqrt{\frac{l}{g \sin i}}$$

in which T_i is the period of oscillation of the pendulum for a given position of the axis AA'; whenever i diminishes, the period increases and theoretically should even tend toward infinite when the axis AA' tends toward a vertical position.

By contrast, when i = 90°, the pendulum becomes an ordinary pendulum swinging in a vertical plane and whose period amounts to

$$T_0 = 2\pi\sqrt{\frac{l}{g}}$$

In the above expressions, l is obviously the reduced length or equivalent length of the single pendulum.

Whenever i is very small, the Mach pendulum becomes a model of the horizontal pendulum from which we shall deduce some of the characteristics.

We will note immediately that, for different angles of inclination or i, i', i", etc., we have

$$T_0^2 = T_i^2 \sin. i = T_{i'}^2 \sin. i' = \ldots\ldots$$

so that, if we know T_o and T_i, we can write

$$\sin i = \frac{T_0^2}{T_i^2}$$

Indirectly, this expression gives the possibility of measuring i from the duration of oscillation. We shall also see that it gives us the means of determining the sensitivity of the horizontal pendulum to variations in the direction of the vertical.

For any given position of the frame supporting the pendulum, it is necessary, whenever the equilibrium is attained, that the center of gravity of the pendulum occupies the lowest possible position compatible with the connections. In order to realize this condition, the center of gravity must be located in the vertical plane passing through the axis AA'.

If the direction of the vertical in relation to the pendulum changes for any reason, the latter will perform a given rotation around AA' in such manner that the center of gravity becomes located in the new vertical plane of AA' and this motion may become very large, even for an infinitely small deviation from the vertical, if AA' occupies a position almost coincident with the original vertical. By making AA' almost vertical, it is therefore possible to automatically achieve a very high amplification of the apparent deviations of the plumb line and the Mach pendulum then becomes a horizontal pendulum.

Position of equilibrium of the horizontal pendulum

The movements of the ground caused by the attraction of sun and moon are periodic movements and, strictly speaking, we would have to make a dynamic study in order to examine their influence on the positions of the pendulum. However, since the periods corresponding to the movements of the ground are very long in relation to the characteristic periods of oscillation of the pendulum, we can limit ourselves to a static study of the equilibrium in order to simplify the explanation.

We saw that a change of the direction from the vertical in relation to the frame of the instrument produces a change of the position of equilibrium of the pendulum. As a function of a deviation d from the vertical assumed as known, we shall calculate the angular displacement ρ which the pendulum arm will carry out in order to occupy its new position of equilibrium.

It is evident that only the displacements relative to the vertical need to be considered; these displacements may originate both by a real change in the direction of gravity and from an instability of the ground on which the instrument rests.

Let us suppose (cf. Fig. 7) an ideal pendulum in which the arm b has a negligible mass in relation to the terminal spherical bob m whose center G may be assimilated to the center of gravity of the pendulum. Let us consider a sphere passing through the center of the bob where the center of the former is located at the point of encounter O of the axis of rotation AA' of the pendulum and of the arm b.

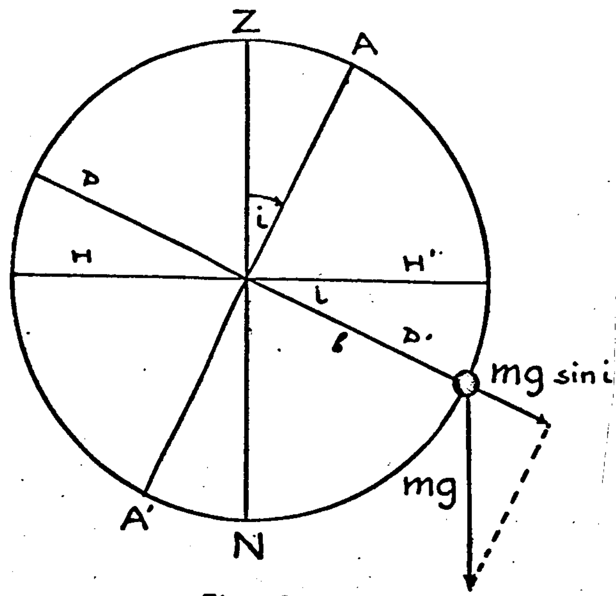


Fig. 7.

The points Z (zenith) and N (nadir) are the points of passage in the respective spherical surface of the vertical of the point O. The axis AA' and this vertical form a very small angle i and this angle characterizes the sensitivity of the ideal pendulum as we shall see. In the figure, the plane of oscillation of the pendulum is the plane PP' seen from the side; it forms an angle i with the horizontal plane HH'; the component mg sin i of the force of gravity located in the extension of the pendulum arm assumed to be in equilibrium and applied to the center of the mass m is shown in the figure. In view of the smallness of i, this component maintaining the pendulum in equilibirum is very small; accordingly, in practice, the slightest disturbance such as a small air draft may disturb the equilibrium so that it is necessary to enclose the pendulum in a small hermetically sealed casing protected from all variations of temperature.

If the pendulum is moved from its position of equilibrium by an angle ρ, there is created a compensating force of gravity whose moment $\mathfrak{M}_p$ amounts to

$$\mathfrak{M}_\rho = m \cdot g \cdot l \cdot \sin.\ i \sin \rho$$

in which l is the length of the segment OG. In view of the smallness of the angles i and ρ, we can write without any appreciable error

$$\mathfrak{M}_\rho = m \cdot g \cdot l \cdot i \cdot \rho$$

in which i and ρ are in radians.

The compensating force is proportional to the elongation ρ; this gives us a case analogous to that of the Mach pendulum discussed previously and also from the point of view of the duration of the oscillations, in the case of a vertical pendulum of length l oscillating in a field of gravity of low intensity where the acceleration amounts to only g x i. The period T_i of oscillation will be, as in the case of the Mach Pendulum,

$$T_i = 2\pi\sqrt{\frac{l}{g i}}$$

If it were possible to rectify the pendulum without breaking the suspension wires and without modifying the position of the point O on the arm b, the period of oscillation of the pendulum now vertical would be

$$T_0 = 2\pi\sqrt{\frac{l}{g}}$$

The values T_i and T_o thus determined would permit us to calculate the angle i whose significance in characterizing the sensitivity of the pendulum will be demonstrated further below.

Since it is obviously impossible to rectify a bifilar suspension as specified above, the designers provide on the arm b a knife-edge whose edge passes through O. It is thus possible to suspend the pendulum by means of the knife-edge from a sort of stirrup and to let it oscillate in a vertical plane in order to measure T_o. This method is not very accurate because it is almost impossible to produce a strict coincidence of the edge of the fulcrum with the point O; on the other hand, the pendulum arm is weighted down by secondary devices whereas it should be kept as light as possible.

We shall now calculate the sensitivity of the pendulum and determine the deviation of the arm b as the consequence of a variation in the direction of the vertical.

Let us assume that the vertical of the locus passes from OZ to OZ' (cf. Fig. 8) and that, in that case, the zenith passes along the elementary arc ZZ' = d along a large arc of circle whose plane is normal to the pendulum arm.

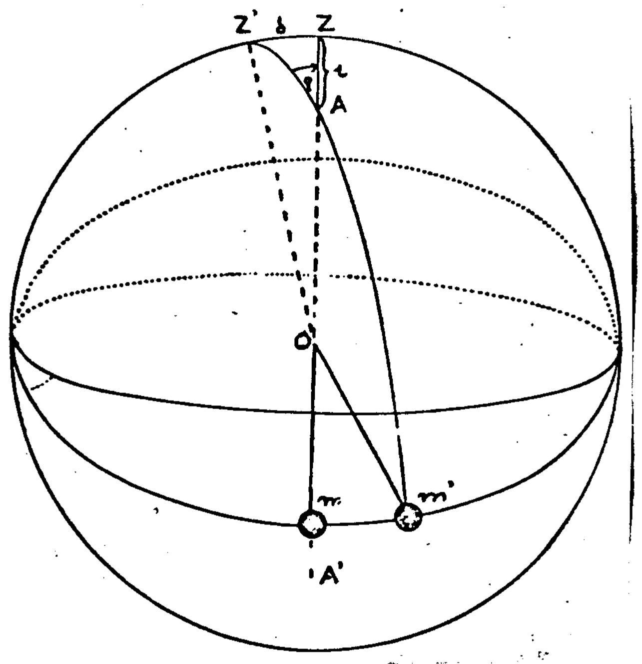


Fig. 8.

Under this influence of the deviation from the vertical, the pendulum is disequilibrated; in order to return to a stable equilibrium, the bob becomes displaced from m to m' and its center will be located in the new vertical plane of the axis AA'.

The arc of the large circle Z'A defines this new vertical plane of AA' on the sphere; the extension of Z'A consequently passes through the center of m'.

In the elementary rectangular triangle ZZ'A with right angle at Z, there appear as elements the arcs d and i and the angle ρ so that we can write

$$\sin. i = \text{tang. } d \text{ cotg. } \rho$$

or, in view of the smallness of the angles,

$$\rho = \frac{d}{i},$$ in which ρ is given in radians.

The angle i characterizes the sensitivity of the ideal pendulum; the smaller i, the greater the deviation of the pendulum arm due to a given deviation d from the vertical.

The factor 1/i thus characterizes the amplifying power of the pendulum; if we are able to give i the value 1", the coefficient of amplification would become 206,265.

In the above explanation, we were always speaking of an ideal pendulum because we assumed a theoretical axis of rotation not opposing any resistance to the angular displacement of the pendulum.

Actually, we must take into consideration the rigidity of the suspension wires because, in case of imperfect construction which is frequent in practice, the torsional resistance of the wires may reduce the sensitivity of the instrument to such a point as to make it unsuitable for any serious scientific use. In the following pages, we shall examine this important questions of the rigidity of the suspension and attempt to show how, by an appropriate selection of geometrical dimensions and the substance of the wires, we can prevent any high torsional resistance and still retain sufficient resistance to rupture.

Calculating the influence of the suspension wire on the value of the coefficient of amplification

We know that the coefficient of amplification of the ideal pendulum is given by the reciprocal of i if this angle is expressed in radians; the geometrical dimensions and the mass of the pendulum do not enter into the calculation of this coefficient.

On the other hand, in the case of an actual pendulum where the characteristics of the suspension wires are a factor, we must take into account various factors, in order to obtain the coefficient of amplification, such as the dimension of the wires and the qualities of the substance of which they are composed, the mass of the pendulum and the position of its center of gravity in relation to the imaginary axis of rotation AA'.

In that case, the coefficient of amplification becomes

$$\frac{I}{i + \varepsilon}$$

in which i is again the angle formed by AA' with the vertical, and ε is an angle whose value is a function of the various factors listed above as we shall see.

In order to calculate the position of equilibrium of the pendulum in this somewhat more complex case, it would be well to consider separately the three elementary forces acting here:

1) A force which we shall call driving force and derived from the deviation of the vertical to which the pendulum is assumed to react;

2) a force compensating gravity and already considered in the preceding paragraph;

3) a compensating force additive to the foregoing and derived from the torsional resistance of the suspension wires.

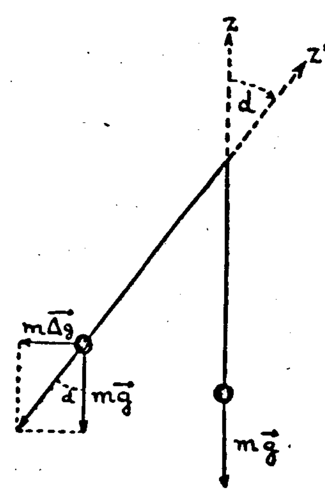


Fig. 9.

From a didactic point of view, the following observation may be useful in regard to the force considered under 1) above: any deviation of the vertical with an angle v which shifts a freely suspended mass m from position P to position P' may be considered (see Fig. 9) as deriving from an elementary force $m\overrightarrow{\Delta g}$ shown as a vector in the figure which is added to the original force of gravity $\overrightarrow{mg}$ applied at the center of gravity of the mass. In that case, we have

$$\operatorname{tg} d = \frac{\Delta g}{g}$$

This is the elementray force which, in the horizontal pendulum, creates a rotary torque which deviates the pendulum arm.

If l is the length of the segment limited by the point O and the center of gravity G of the pendulum arm, the moment of this elementray force around O may be written as

$$\mathfrak{M}_d = m\,l\,\Delta g = m\,l\,g\,.\operatorname{tg} d \sim m\,l\,g\,d$$

The couple thus defined is a deflecting torque but there are two resisting couples which slow down rotation:

The first of these is the restoring torque of gravity whose moment around O is equal to

$$\mathfrak{M}_P = m\,l\,g\,i\,\rho$$

and has already been considered in the preceding paragraph.

There is also a restoring torque due to the torsion of the wires whose moment around O can be represented, in accordance with the laws proposed by Coulomb, by

$$\mathfrak{M}_T = \eta\,\rho$$

in which η = sum of the modulus of torsion of the two suspension wires; we shall further below explain how to calculate the value of η. η is a constant value depending on the nature of the wires (modulus of rigidity and geometric characteristics $\eta = \mu \frac{\pi r^4}{2l}$).

In order to obtain an equilibrium of the pendulum, the torque must be equal to the sum of the moment of the two resisting couples and consequently

$$m\,l\,gd = m\,l\,gi\,\rho + \eta\rho$$

or also

$$\rho = \frac{mg\,l\,d}{m\,l\,gi + \eta} = \frac{d}{i + \frac{\eta}{m\,l\,g}}$$

We state

$$\frac{\eta}{m\,l\,g} = \varepsilon,$$

in which ε = a given angle whose value is expressed in radians and consequently

$$\rho = \frac{d}{i + \varepsilon}$$

In this expression, i is still the angle between the imaginary axis of rotation AA' with the vertical and ε is an angle whose value becomes as much greater as the quality of the pendulum wires is less.

In the case of the ORB pendulums, we shall subsequently see that the angle ε here expressed in seconds of arc is an average 4.5". The value of the angle $(i + \varepsilon)$ now characterizing the sensitivity of an actual pendulum can be easily measured, for a strictly determined period of oscillation T, by means of an expansible crapaudine (cf. Chapter 2). For T = 80 sec, we find $i + \varepsilon$ = 10". The coefficient of amplification of the single pendulum under these conditions is 20,626. However, in practice we must take into account the supplementary amplification resulting from the application of the optical methods of the so-called rotating mirror of Poggendorf. The physical arm of quartz whose length is about 10 cm then is replaced by a pencil of light with a length of 5 m whose efficiency must be multiplied by a factor of 2 by reason of the laws of reflection. The amplification of the optical method then becomes about 100 and the total amplification of the photographic recording is about 2×10^6. In that case, the movement of the end of the pencil of light which scans the photographic film of the recorder, has amplitudes equivalent to those of the movements of the end of a freely suspended vertical pendulum which would be more than 200 km long.

CONSTRUCTION PRINCIPLES OF ORB PENDULUMS

Before explaining the construction principles of the ORB pendulums, it will be necessary to demonstrate the extraordinary smallness of the relative movement of the ground and of the vertical which these instruments must detect and measure accurately. The following considerations would seem of a kind to clearly show how strictly stable the pendulum itself must be so that purely instrumental errors will not obscure the geophysical phenomena which are to be determined.

The relative deviations of the ground and of the vertical are generally inferior to 0.025" in their largest amplitude.

For the ORB pendulum where the small sides of the rectangular triangle of the base have about 0.25 m, this angle corresponds to relative variations of less than 0.025 micron in height of the 3 points on the ground on which the 3 set-screws rest.

Actually, 0.025 micron is relative to the largest amplitude of the phenomenon and we must measure fractions of this value on the photographic recording of the motions of the pendulum. Consequently, a variation of ordinate of 1 mm which can be read from the recording practically with the naked eye, originates on the average from a deviation from the vertical not greater than 0.001" and this corresponds to differences of level of the base point of about 1 millionth of 1 mm or 10 Å!

10 Å is on the same order of magnitude as the molecular unit of the quartz (cf. Fig. 10).

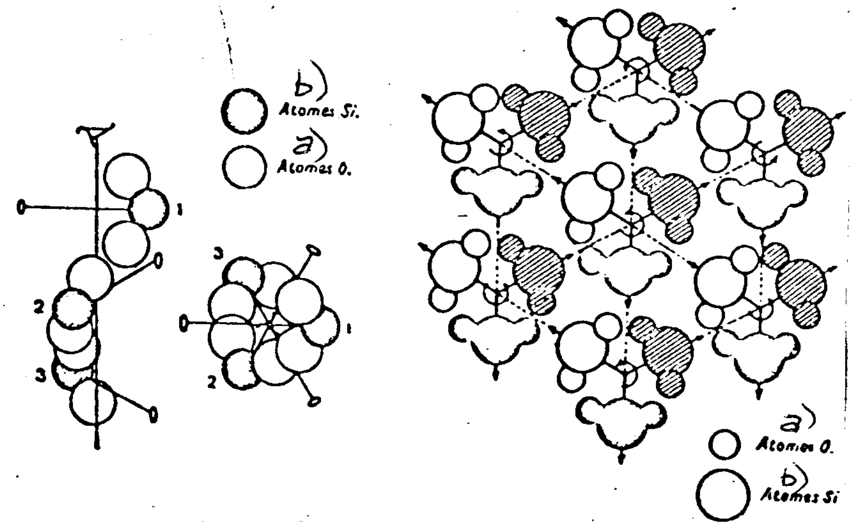


Fig. 10 - Diagram of molecular distribution in a quartz crystal. The unit group is formed by 3 SiO_2 molecules arranged in a spiral and forming a cell or unit mesh with a height of 5.37 Å (cf. left figure). The right figure shows the arrangement of the cells in a crystal.
Legend: a = oxygen atoms; b = silicium atoms.

It is noteworthy that we are confronted here, from the point of view of these still perceptible movements, by distances which are below the separating power of standard electronic microscopes. The above considerations are of a kind to make any researcher hesitate who is attempting to construct a horizontal pendulum and make him aware that all parts of the instrument must have a form appropriate for maximum rigidity as well as that the material for construction must have as low a thermal coefficient as possible and must moreover not be subject to internal variations due to slipping or dislocation which may suddenly take place in the crystalline structure of the metal.

Now then, when we construct a metallic pendulum, it is very difficult to cut the suspension wires or bands to the exact desired length. Consequently, it is necessary to introduce, at the points of fixation, adjusting screws (cf. Fig. 14) which actually interfere with the stability of the instrument in time.

Moreover, in the case of bands whose section is 300 x 20 microns, it is difficult to define the point through which the axis of rotation of the pendulum passes. We are not even sure that this point can be defined as such.

"Molten silica" as material for horizontal pendulums

The extreme stability of the following two materials seems to make them singularly appropriate for the construction of such high-precision instruments: i.e., molten quartz or, strictly speaking, molten silica (SiO_2), and a metal alloy discovered by Masumoto (63.5 % Fe, 31.5 % Ni, 5 % Co and traces of Mn) called "super invar" with a coefficient of expansion of less than $10^{-7}/^{\circ}$ C.

Certain properties of silica, specifically the ease with which thin silica wires can be fused autogenously to the quartz frame of the instrument, make silica preferable. We can thus eliminate connecting collars and screws.

Quartz is a crystallized variety of silica and is found abundantly in nature in the form of siliceous sand on the surface of the ground. Heated to 2,000° C, it loses its crystal form and, after cooling, we obtain a vitreous isotropic and amorphous mass. This is silica glass one of whose essential properties is a remarkable insensitivity to temperature variations which not even the original quartz crystal possesses to the same degree.

The coefficient of linear expansion of molten silica is very small within large limits of temperature. Between 0 and 1,000° C, it is on the average 0.54×10^{-6} or on the same order of magnitude as that of super invar which has the great disadvantage that its minimum coefficient is not attained except within very narrow limits of temperature.

Silica glass is extremely hard, cuts all other types of glass and has a low specific weight with a density of 2.21.

Another particularly important quality of silica glass, especially in regard to suspension wires, is the fact that its viscosity is nearly zero. If a silica fiber is twisted, there is practically no loss of energy through intermolecular friction whereas this is the case for all metals. With a suspension of thin silica threads, it is possible to construct an instrument returning to zero in a highly satisfactory manner and presenting remarkable accuracy due to the absence of any appreciable hysteresis. This is why silica threads have been used for many years in the suspension of the frames of very high-precision galvanometers. Very thin silica wires have a remarkable rupture strength which is greater than that of most metals except tungsten.

Properties of hot-drawn silica thread

It is very easy to produce in the laboratory very great lengths of accurately calibrated silica thread by drawing the wire from a cylindrical rod which is brought to incandescence by an oxyhydrogen flame and by winding the wire on a uniformly rotating disc. We shall not here go into the technical details of this process but simply examine the characteristics of such threads.

a) Rupture strength.

This is one of the essential characteristics with which we must be familiar before constructing a suspension.

Scientific literature on the subject is extensive but the data furnished by various authors differ slightly.

As an example, Fig. 11 shows the curve of rupture strength in gram and as a function of the diameter of aged threads which was plotted from values found experimentally by Reinkober (Phys. Zeit., 1938, p. 112).

Reinkober indicates that the rupture strength diminishes slightly for a certain time after drawing. We always accept the values of Reinkober but do take into account for all suspensions a coefficient of safety of 3 so that a 40-micron thread such as utilized for the suspensions of the ORB pendulums is considered adequate to support a load of 20 g.

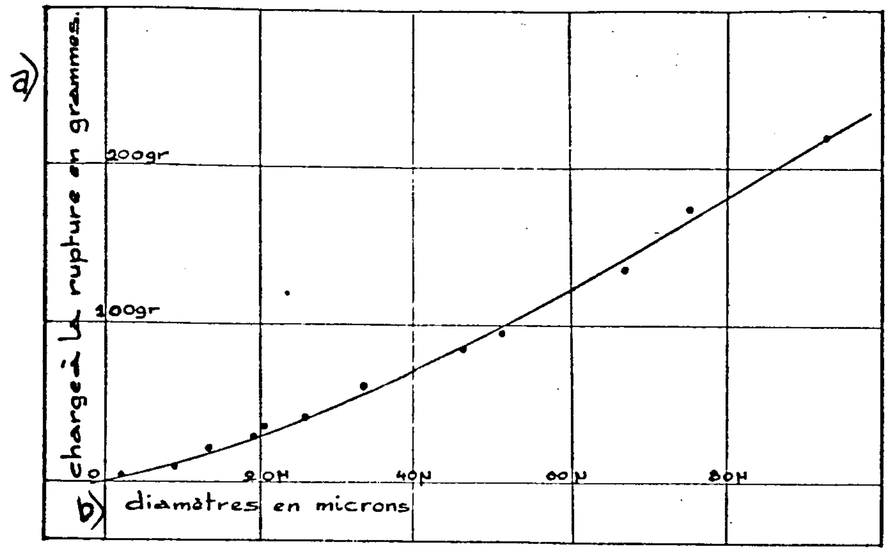


Fig. 11 - Graphic of rupture strength of thin silica threads. Solid points correspond to the values found experimentally by Reinkober. It will be noticed that the rupture strength diminishes much slower for small diameters than it would if the cross-section of the thread alone was taken into account.
Legend: a = rupture load in g; b = diameter in microns.

b) Arrangement of ORB pendulum suspension wires.

Fig. 12 shows, at a scale of 1 : 2, the arrangement of two suspension wires in relation to the pendulum arm.

The points of connection at the frame A and B define the position of the practically vertical imaginary axis of rotation AOB. Consequently, the quality of the connections A and B plays an essential role and is much more important than the quality of the connections at C and D on the pendulum arm itself. As a function of design, the connection C is in the middle between the connection D and the center of gravity G of the arm. In addition, the plane of oscillation of the pendulum is located half-way between the points A and B so that we have AO = OB where O is the center of oscillation of the pendulum.

c) Thread tension and angles α and β of the latter with the vertical.

The pendulum is in equilibrium in a vertical plane due to three forces shown as vectors in the figure: the force of gravity $\overline{P}$ applied to the center of gravity G of the arm, the tension $\bar{a}$ of the upper wire and $\bar{b}$ of the lower wire.

The relations of equilibrium are:

$$a \cos \alpha = 2P = 2b \cos. \beta$$
$$a \sin \alpha = b \sin. \beta$$

from which we derive

$$\tan g\, \alpha = \tfrac{1}{2} \tan g.\, \beta.$$

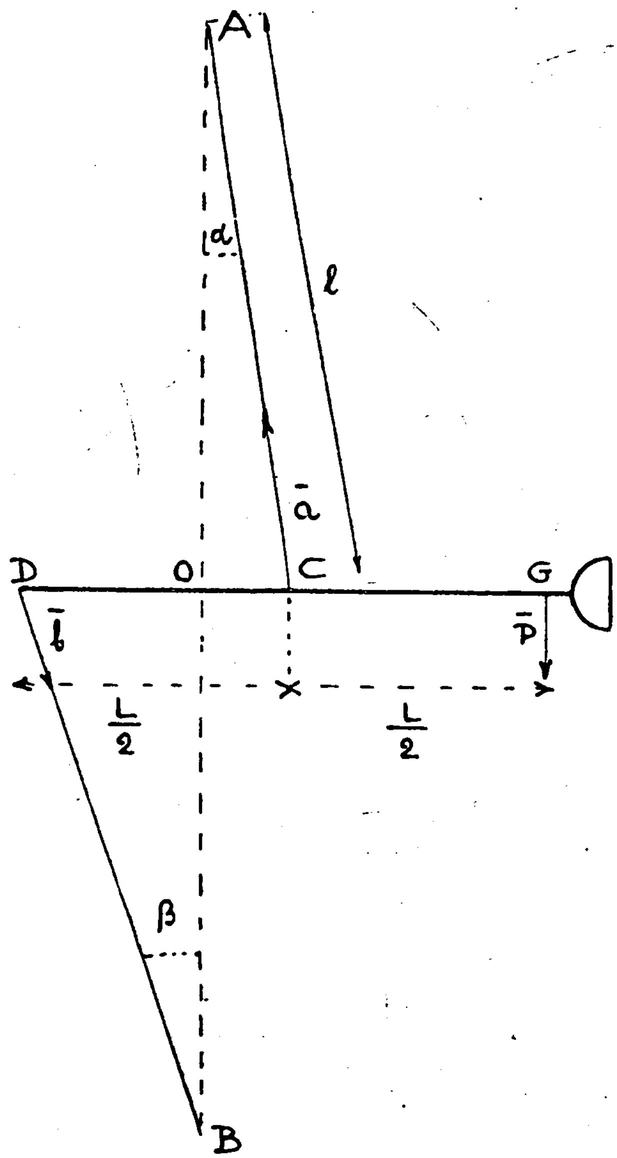


Fig. 12 - Suspension of ORB pendulum, scale $\frac{1}{2}$.

Since the segments OA and OB are equal, we deduce from this that OC = 2OD so that the center O is located at one-third of the length on the segment CG.

We further derive from these relations:

$$\sin \alpha = \frac{1}{6} \frac{L}{l}$$

$$\sin \beta = \frac{1}{3} \frac{L}{l}.$$

For the ORB pendulums, the length l of the suspension threads is 10.5 cm and the length L of the segment CG is 11.0 cm so that we can calculate

$$\alpha = 10^\circ \quad \text{and} \quad \beta = 20^\circ$$

For evaluating the tensions a and b, we can take the cosines as equal to unity by reason of the low value of the angles α and β. We have

$$a = 2P = 20\,\text{gr.} \quad \text{and} \quad b = P = 10\,\text{gr.}$$

in which P = weight of mobile arm.

d) <u>Resistance of wires to rotation of pendulum.</u>

We have already mentioned the modulus of torsion of the suspension wires of the mobile component. We shall now give some additional details on this and carry out the calculation of the angle ε which basically characterizes the quality of suspension.

If the end of any desired thread is attached invariably, and we wish the other end to rotate through an angle φ, we must apply a torque with a moment designated as $\mathfrak{M}_T$.

Within the limits of elasticity, this moment is independent of the tension of the thread and proportional to the angle of torsion φ so that we can write

$$\mathfrak{M}_T = \eta \cdot \varphi$$

in which η = modulus of torsion of thread. This is a constant determined by the geometric dimensions and the substance of the thread. Coulomb gave a mathematical expression of η as

$$\eta = \mu \cdot \frac{\pi r^4}{2 l}$$

in which μ is the modulus of rigidity of the substance utilized. The modulus η of torsion of the wire has the dimensions of a moment. This magnitude is numerically equal to the moment of torsion which would be produced by a rotation of the free end of the thread equal to the unit angle or $57^{\circ}18'$ (1 radian), whereas μ has dimensions of a pressure and is a force per unit surface. μ can be expressed in baryes when employing the CGS system.

The value of μ varies, however, with the diameter of the thread when the latter is very thin. Consequently, for threads with a diameter of 40 micron, the value adopted is 3.3×10^{11} (dyne/cm^2). We saw that the suspension wires each had a length of 11 cm. We must now add up the modulus of the 2 threads and this means that we calculate η for a wire 5.5 cm long.

We have

$$\eta = \mu \frac{\pi r^4}{2l} = 3{,}3 \cdot 10^{11} \frac{3{,}14 \cdot 20^4}{2 \cdot 5{,}5 \cdot 10^{16}} = 1{,}5 \text{ dyne/cm.}$$

From the expression $\eta = mgl\,\varepsilon$, we derive m = 10 g and l = 7.0 cm :

$$\varepsilon = 4''\,5$$

Attaching silica threads for bifilary suspension

Molten silica undeniably presents the most favorable qualities for the construction of excellent horizontal pendulums.

However, designers hesitate to utilize it because of the difficulties inherent in working with quartz at high temperature.

Dr. Anton Graf writes: "Working with quartz is appreciably more difficult than working with glass. Quartz has only a very small range of fluidity and becomes sublimated even at a low excess of heat. Amazing manual skill and years of practice are required to succeed in the construction of the first successful measuring system."

We entirely agree with Dr. Graf in regard to the quartz tubing of certain complicated chemical apparatus which are masterpieces of manual skill. The tripods of the ORB pendulums are constructed in this manner by a specialized company at Brussels from tubes with a diameter of about 6 mm skillfully joined to each other. It would be much easier to construct such a tetrahedron with solid rods. However, for the same weight, it would be much less rigid and elegant.

However, there is one operation of working with quartz which we must be capable of executing ourselves before installing a horizontal pendulum of this type in an underground mine and this is the replacement of an accidentally broken suspension thread.

At first sight, it would seem that such replacement of a thread is more difficult than any other such operation. Since there is the risk of sublimation, this risk is ever so much more increased when we must hold in the flame of an oxyhydrogen torch an almost invisible thread which floats with the air currents and has a diameter of only a few microns.

With certain precautions and by making use of a particular property of molten silica to which we shall return further below, it is actually possible to carry out such a replacement without risk in a few minutes. Once installed, the pendulum will no longer occasion the slightest difficulties in regard to the silica-thread suspension because the latter appears capable of lasting a great number of years except for a somewhat heavy earthquake.

Working with the blow torch

The fusion temperature of silica is in the neighborhood of that of platinum (1,700°). In order to fuse silica, we must therefore utilize a high-temperature torch powered by a mixture of oxygen and hydrogen or butane, etc.

Concerning the fusion of thin threads, the temperature of the flame must be carefully controlled to prevent sublimation. The oxyhydrogen flame should not be used because the wire breaks either due to excessive sublimation or excessive force of the gas issuing from the torch jet.

The mixture of oxygen and butane has produced better results. In addition, butane has the advantage of being available everywhere in small bottles which can easily be transported in underground caves or mine galleries. For working the threads, the aperture of the torch jet should have a diameter of about 0.2 mm but pressure must be reduced so that the flame will be short and soft (not issue with excessive force).

Surface tension of liquified silica

Molten silica has a very high surface tension which may rise to as much as 250 dyne/cm with a favorable temperature.

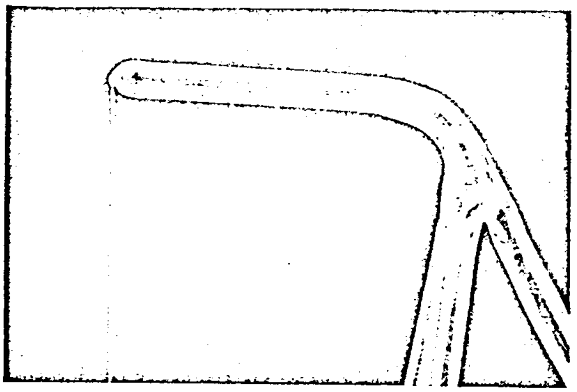
This tension is the cause of certain particular phenomena observed when silica threads are heated with the torch to fusion temperature.

If we suspend a thin thread of silica 10 cm long vertically and bring it to incandescence until fusion a little below its point of junction, we will note that the surface tension overcomes gravity and that the whole of the thread is drawn up toward the incandescent part which becomes thickened. Under these conditions, the thread does not break provided that the flame of the torch is well adjusted because there is a greater replenishment of matter by contraction of the thread than a loss of matter by sublimation.

The entire technique of installation of a thread is based on this particular property. It makes it possible to tighten a thread fixed at both ends. If the thread is not too loose, a brief contact with the flame of the torch at one or the other points of junction is sufficient to tighten the thread abruptly.

Construction in practice of a silica-thread suspension

A geophysicist desiring to install a horizontal pendulum of molten silica in an underground cave should be capable of himself repairing a suspension broken either accidentally or by a somewhat heavier earthquake.



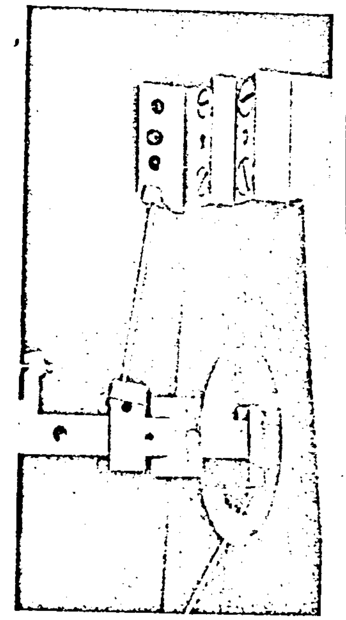


Fig. 13 - Autogenous weld of quartz thread to quartz housing.

Fig. 14 - Screw-clamp connection of suspension band (phosphor bronze) to metallic frame (due to a luminous phenomenon, the threads appear much larger in these figures than they are in reality). →

We are therefore explaining below the technique of replacing the suspension wires of the arm. Here nothing quite takes the place of direct observation of the work of a specialist but, since this is not always possible, we hope that the following description on handling the silica threads will be such as to make possible "self-training."

The first operations are performed simply by utilizing two small silica rods a few centimeters long and held in certain cases on an appropriate support which is easy to construct (cf. Fig. 15).

FIRST OPERATION

Remarks on fusion of silica

The torch used (cf. Fig. 16) is small. To begin with, we must acquire a certain skill in accurate adjustment of the high-temperature flame powered by a mixture of either hydrogen and oxygen or butane. Success of the experiment depends largely on proper adjustment of the flame.

The oxyhydrogen flame is suitable for welding relatively large parts but should not be used for thin threads because its action is too intense and

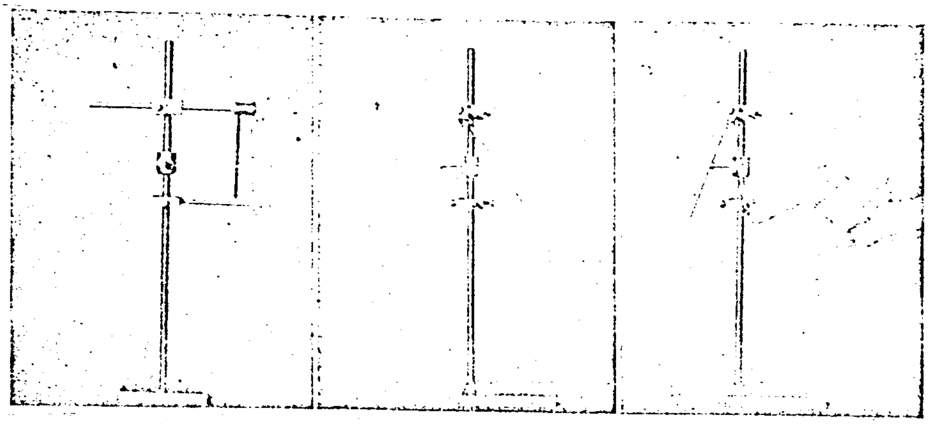


Fig. 15 - Simplified support providing handling in height and inclination of cylindrical quartz pieces.

The support is utilized either for facilitating the attachment of thin suspension threads of a horizontal pendulum or during manipulation III.

A vertical shaft A carries a rectangular piece p (upper part) which slides with a slight degree of friction on A; the screw v locks p on A in the desired position. A horizontal shaft hh' slides with a slight degree of friction in a barrel drilled laterally into p; hh' has at one end a screw-clamp allowing us to fix, in a position perpendicular to hh' as will be seen from the figure, the pendulum arm which is to be provided with the suspension.

We also see a second rectangular piece p' (lower part) which slides on A like the first and in which a lateral barrel has also been drilled in which a cylindrical rod slides. The figure on the right clearly shows how we can communicate to the pendulum arm a limited inclination with accuracy and adjustable amplitude by pressing on one of the two levers in contact with the horizontal rod serving as stop. Two additional stop rings can be placed at the top and the bottom on A in order to appropriately limit the travel of the moving parts (cf. Fig. 19). Between the moving parts, we also see a stop which is adjustable and carries on its upper face a coil spring with a few turns. During manipulation III, the support is used by simply sliding the two pieces of quartz which are to be joined by thread in the barrels already mentioned.

causes breaking of the threads. A mixture of oxygen and butane is preferable because its action can be regulated more carefully. By varying the pressure of the gases (mixed gases), it is possible to adjust the intensity of the flame. In order to work on thin threads, the flame should not have too much pressure but should "float" at the jet of the torch and should be "soft", rather large at the base, and not have the customary pointed appearance of the oxyhydrogen flame. The oxyhydrogen flame is nearly invisible except in a half-light whereas the oxybutane flame shows a good deal of color and a characteristic internal texture. It has a generally pale-blue appearance except directly at the jet of the torch where there is a relatively pointed and more luminous cone which forms a sort of core within the flame. The heat is highest at the summit of this central cone (cf. Fig. 16).

Let us now observe the fusion of the quartz in this flame. The end of a thin rod is plunged into the flame near the summit of the central cone referred to above. We will see almost immediately a lively incandescence which produces itself close to the point of fusion (1,700° C). The end of a second rod is placed into contact in the flame with the end of the first rod. Fusion occurs almost immediately and by drawing them out quickly a relatively long and very tenuous thread is obtained (Fig. 17).

NOTE: Silica in fusion emits ultraviolet radiation so that the operator should wear protective goggles.

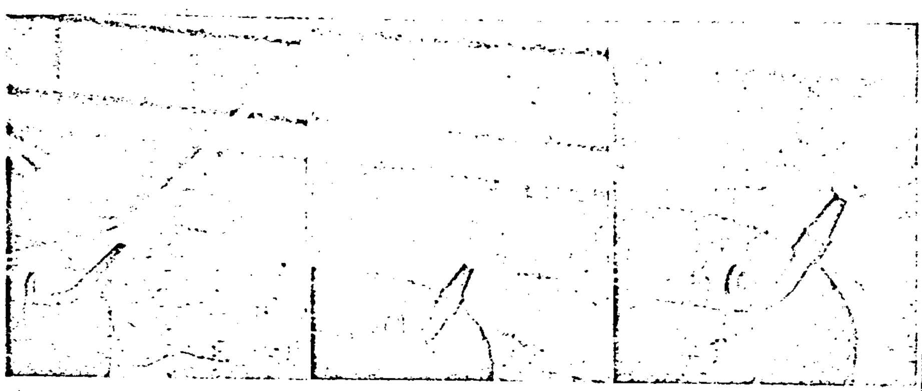


Fig. 16 - The 3 figures show respectively: (a) the hot point of the oxy-hydrogen flame. In order to make the flame sufficiently actinic, it was colored by traces of sodium; (b) natural color of hot point of butane-oxygen torch. The figure shows a flame which is too active for working with thin threads; (c) soft and floating flame due to reduced pressure of the oxygen-butane mixture which is very suitable for working with thin threads.

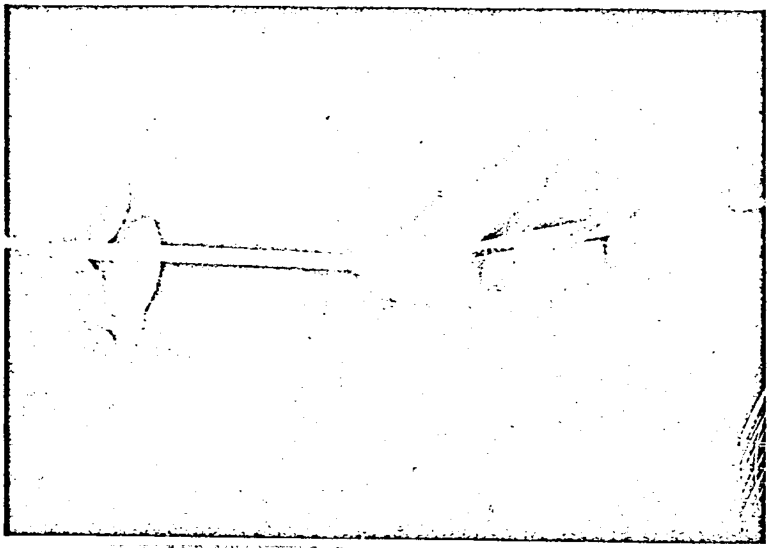


Fig. 17 - The ends of 2 silica rods are confronted in an oxygen-butane flame. Due to internal reflection and refraction of the light from the fused ends, the rods appear to be strongly illuminated over their entire length. Incidental remark: The remarkable properties of quartz threads were only discovered by C. V. Boys around 1889.

SECOND OPERATION

Drawing uniform long threads

We perform the same operations as above but draw the silica rods to obtain a thin thread with a length of 5-6 m. The ends of the two rods having first been brought into contact at the summit of the central cone of the flame, a second operator takes one of the rods and walks away from the first operator. During drawing, the end of the stationary rod is kept at high temperature by remaining partially inserted in the flame as indicated in Fig. 18.

The thread produced will be as much thinner as the speed of drawing has been greater. After some experimentation, we easily obtain threads with a diameter of 40 μ which can be used for the suspension of ORB pendulums.

At the present time, we actually produce in the laboratory continuous threads several hundred meters long by substituting for the second operator a uniformly rotating drum on which the thread is wound at constant speed (cf. Fig. 18). Under these conditions, the uniform speed of drawing results in a uniform diameter of the thread over very great lengths.

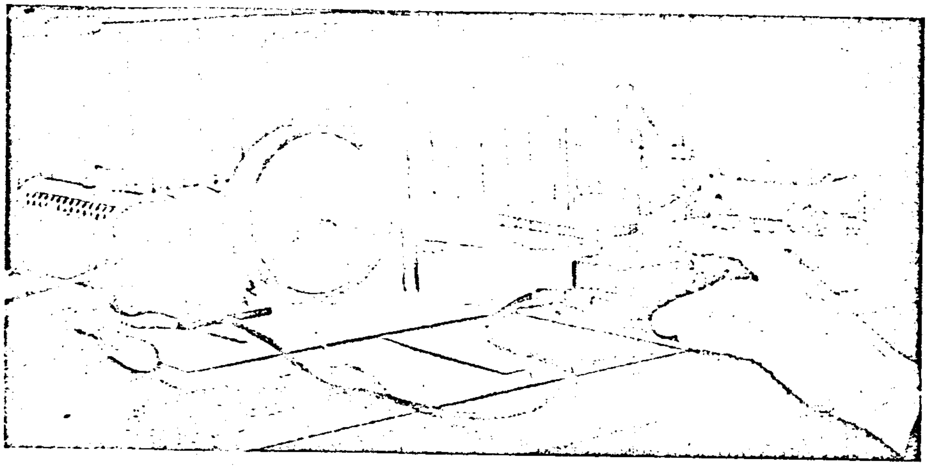


Fig. 18 - Continuous drawing of thread from a quartz rod with one end kept in fusion in an oxyhydrogen flame. The operator activates the switch controlling the rotation of the disc with the left hand while operating with the right hand a screw which feeds the rod slowly into the flame for the purpose of gradually compensating the loss of matter constituted by the thread wound on the disc.

THIRD OPERATION

Junction by autogenous welding

a) From the end of a thin thread to a silica part;

b) Placing a taut wire between two fixed points.

These operations are carried out with two small rods held in parallel position by means of the support (Fig. 15).

a) Welding end of thin wire to a silica part.

At the desired location, we first draw out a thin filiform appendix. This is done as follows: Preferably using a very diluted oxyhydrogen flame (mixture of hydrogen and oxygen, aperture of jet less than 0.1 mm), a very small surface at the desired location on the part to which the wire is to be attached, is brought into fusion. This incandescent point is brought into contact with the extremity of a thin rod (diameter 2 mm). The latter will immediately adhere and is quickly drawn away in a thin thread which is cut off with the torch to leave a section about 1 cm long.

The end of the calibrated wire will then be welded to this section as follows:

The end of the long wire is placed slightly crosswise on the end of the section and rapid contact with the flame of the torch almost instantaneously produces fusion and an autogenous weld between the joint ends of the thread and the section.

We now need only make the junction uniform which is not difficult if we use the high surface tension of fused silica for this purpose.

The thread is held by the left hand at the lower end and the flame is again directed on the point of junction. As soon as fusion temperature is reached, capillary tension sets in and the wire is drawn toward the liquified matter.

In order to prevent gravity from overcoming attraction if the thread is very long, it must be gradually released from the hand at the rate by which it is absorbed in the mass. As soon as the operation seems completed, the flame must be quickly withdrawn at the same time as we exercise a slight tension on the thread in the direction in which it is normally to be stretched. Under the effect of this tension, the section still in the process of fusion at the point of junction will slightly stretch and the junction will be surprisingly uniform.

b) Placing a taut wire between two points.

At first sight, it would seem that installation of the thread would require only repetition of the same operation as above. Actually additional difficulties are inherent in this operation because the thread must be tightened between the two points of connection.

The two operations described below show the procedure.

(1) The two parts to be connected are placed on the support and, during a first test, we assume that the operator is free to slide on A the upper moving

block to which is fastened the part p whereas the lower part p' is assumed to be stationary on the axis during the operation.

We first connect the two parts by a loose thread appreciably longer than the distance between the two pieces. The operation is simple because, in view of the excess length of the thread, the latter can be handled during both operations as if one of its ends were free.

There remains the final operation of tightening the thread between the two points of connection.

In principle, it is sufficient to bring to fusion temperature a short length of the thread so that surface tension will shorten the latter by slightly thickening the points where fusion has taken place.

Actually, some care must be exercised. If we operate with the flame on a completely slack thread, a pocket is rapidly formed by the action of the blast and the thread breaks at the bottom of the former. In order to avoid this, it is always necessary to work with an only slightly slack thread. We must perform this operation gradually by slightly tightening the thread with the hand in such manner that only a slight sag remains during the operation.

With the appropriate support (Fig. 15), the operation becomes simplier because we need only raise the upper moving part to produce a slight tension on the thread. We then direct the flame of the torch on the lower junction and let the upper part slowly slide downward in order to produce the specified conditions. At the instant where the sliding part reaches the stop ring, we almost immediately withdraw the flame by leaving only just enough time for the thread to become completely tightened or 2-3/10 sec.

(2) Two parts with fixed distance.

The foregoing is sufficient to explain how such a junction is made. After having joined the two points by a loose thread, the thread is supported by the hand so as to hold it tight close to one of the points of connection. It is then gradually released from the hand so that the thread always sags only slightly toward the side where the junction is being made.

When the thread is practically tight, tension can be improved by utilizing a short coil spring resting on the stop ring. With the thread almost fully tight, a certain sag will appear again if the spring is slightly compressed.

This sag can be again removed by briefly using the flame on either one or the other junction. At the proper moment, the flame is withdrawn at the same time as liberating the spring which then exercises traction on the thread. This traction is translated by a slight stretching of the junction in the direction of traction of the thread. The various processes used during the third operation are particularly instructive because they are used in constructing the bifilar suspension of a horizontal pendulum.

FOURTH OPERATION

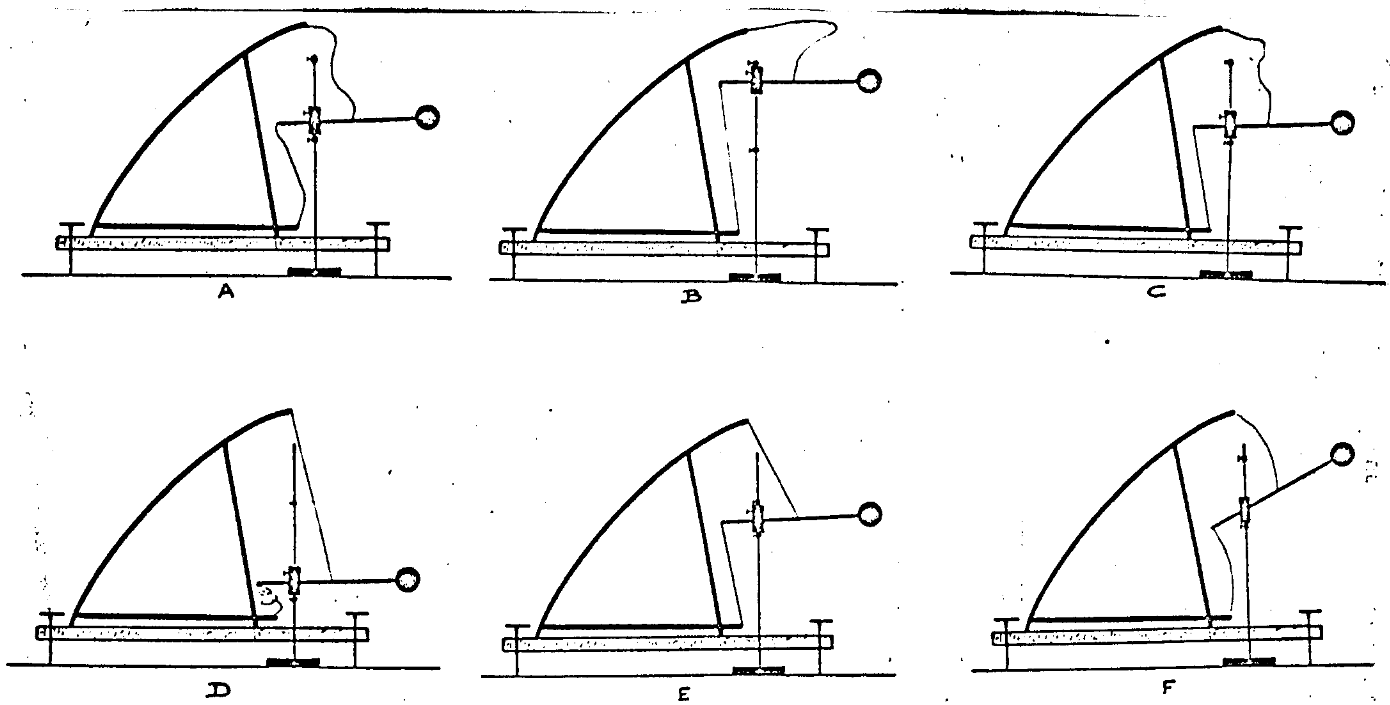
Construction of bifilar suspension of horizontal pendulum

Anyone having successfully performed the various operations described above is qualified for repairing or constructing the bifilar suspension of ORB pendulums.

He will be able by operating simply manually or by utilizing, if he wishes to exercise particular care in making the points of connection, an auxiliary support allowing accurate manipulation of the pendulum arm during the operations.

If the work is performed simply manually, the arm must first be placed in its cradle in the exact position which it normally occupies when it is liberated for use. In order to locate this position, it is sometimes necessary to make two successive approximations for fixing the arm.

We can then proceed to the installation of the suspension in accordance with the method indicated (third operation). The greatest care must be



<u>Fig. 19</u> - The above sketches demonstrate the procedure of utilizing the auxiliary support in order to accurately prepare a bifilar suspension. The tetrahedric frame in the sketches is drawn in solid lines.

A) Pendulum arm placed on the auxiliary support and maintained, in relation to the tetrahedric frame, in the average position which it will occupy in normal service. The two floating threads have been attached;

B) The lower thread is stretched first by sliding the arm of the movable central part upward and the latter can slide on the vertical shaft A;

C) Thermal reduction of the length of the lower thread by heating one or the other point of attachment of the thread and utilizing for this surface tension. During the operation, the movable part with the arm is supported manually and slid downward at the rate of absorption of the excess length of the thread;

D and E) Analogous operations at B and C but on the upper thread. During all these operations, it will be noted from the sketches that the stop rings are appropriately placed in order to limit the travel of the moving part in a suitable manner;

F) Completion of the point of attachments: inclination of the arm allows us to again slightly slacken the threads in order to complete the 4 points of attachment by repeated fusion and traction. (cf. analogous operation, manipulation III, use of coil spring).

exercised in making the two connections to the stationary tetrahedral support itself because they are the ones which define the axis of rotation. The quality of the connections to the arm is less important so that these two can be made after the point of connection to the tetrahedron. It is only after making the first point of connection that it is possible to exercise the slight manual traction which produces a very slight stretching of the junction while orienting this effort strictly in the direction of the tension of the thread whereas the second point of connection cannot be as carefully adjusted.

If greater perfection is desired for all four points of connection, we must use the special support as a manipulator which makes it possible to impart accurate movements to the suspension arm during the operation.

An examination of the sketches of Fig. 19 fully explains the operating procedure for utilizing the support described in Fig. 15.

THE PENDULUMS

Type I (Fig. 20)

They each consist basically of a base of "duralinox" (27 x 27 x 2 cm; weight 4.7 kg) equipped with three vertical set-screws located at the right angle (pitch 0.5 mm). This arrangement makes possible increase of sensitivity to high values and easy and accurate correction of drift during adjustment. Consequently, the three set-screws each have a strictly defined function as has already been indicated. The one which, with the right-angle screw, defines a direction parallel to the pendulum arm is utilized to place the pendulum "in period". The one which defines the perpendicular direction is utilized to regulate the azimuth of the arm and is called "drift screw". It is apparent that the right-angle screw fulfills both these functions so that it is never utilized except for very rough adjustment during installation (elementary leveling of the instrument).

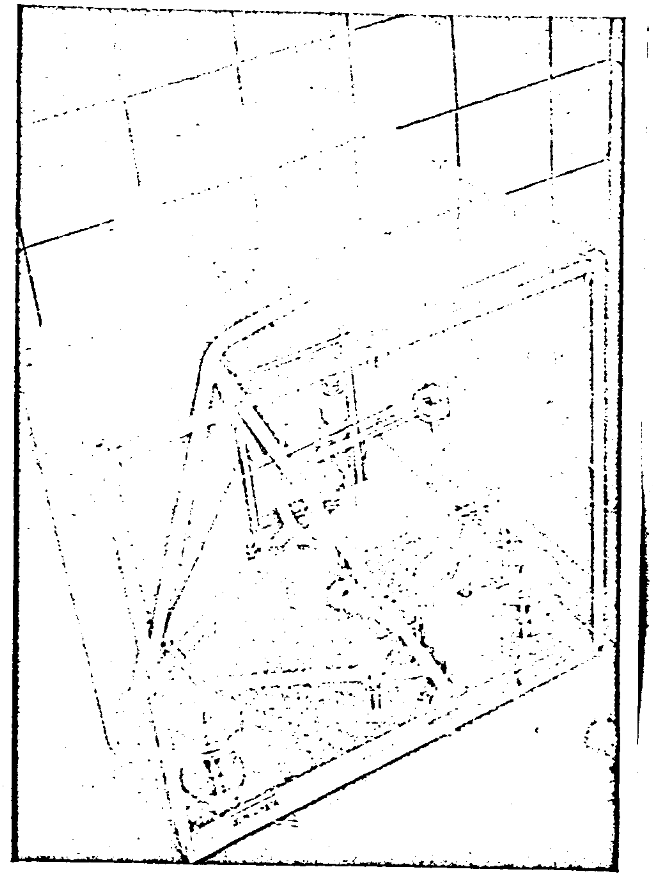


Fig. 20 - ORB pendulum 1, prototype of series.
(Cliché Revue "Industrie")

Fig. 21 - Construction of quartz tetrahedron in the laboratory.

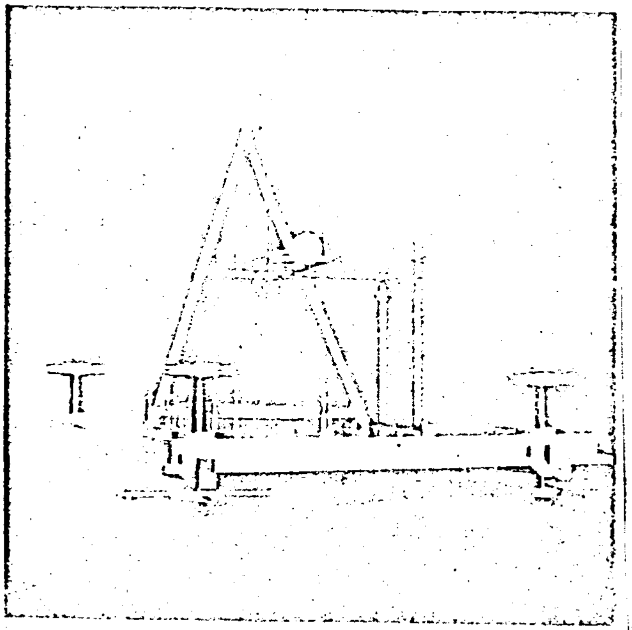


Fig. 22 - ORB pendulum 5 (type I). The pendulum locking cradle can be seen very clearly.

By means of quartz tubes assembled by autogenous welding, we construct a tetrahedron which forms the main frame of the instrument (Fig. 21).

The tetrahedron is firmly held to the base by the pressure of six springs. Moreover, the three feet are slightly blocked by small wedges which prevent sliding during assembly. However, they do leave a certain freedom of movement to the tetrahedron so as not to introduce any stress when changing from the temperature of the laboratory to the temperature of the underground station.

The pendulum is suspended within the tetrahedron in accordance with the so-called Zöllner schema. It is also of a single piece of quartz and consists of a hollow arm with a length of 12 cm terminating in a mass of about 10 g of transparent quartz. It is suspended by two quartz threads with a diameter of 40 micron and a length of 11 cm attached to the frame and the pendulum by autogenous welding.

It is necessary to stress the two fundamental characteristics of the tetrahedral frame and tripod. Its form is geometrically indeformable and perfectly symmetrical in relation to the suspension. After welding, the tetrahedron is annealed and subsequently cooled slowly in a furnace for 24 hours. Prior to utilizing it, it is inspected under polarized light to determine whether all internal stresses have been eliminated.

The tetrahedron occupies an overall surface of 185 mm with a height of 250 mm and has a diameter of the constituent tubes of 6.5 mm. A plane mirror with a diameter of 2 cm is rigidly attached to the pendulum in its axis of rotation. By means of a lens with a focal distance of 5 m, it reflects the image of a luminous spot on the horizontal slit of a photographic recorder.

The pendulum rest forms a sort of cradle (easily seen on Fig. 22) which can be inclined and raised at will by means of an adjusting screw. The cradle supports the pendulum arm so as to slacken the suspension threads. The pendulum arm can be fastened to the cradle by spring clips so that the instrument can be transported over long distances without breaking the threads. The cradle also plays a very important role in the installation of the suspension or its eventual replacement after accidental rupture of the threads. In order to attach the thin quartz wires by autogenous welding, experience has shown that it is necessary for the pendulum arm to be supported strictly in its normal position when the instrument is in use on location.

It will be noted on certain photographs of the prototype ORB pendulum 1 (Fig. 20) that the drift screw is constituted by a modified Palmer (self-threading) screw. This type of screw is no longer used because its internal assembly produces an instability manifested in particular by strong drift and abrupt discontinuity in recording. It has been replaced by a standard screw in pendulum 1.

Type II (Fig. 23)

The principles of the instrument have not been modified in the design of the new pendulums of Type II. However, since experience has shown the uselessness in practice of the right-angle screw which has already been pointed out in theory, this screw has been replaced by a steel foot. This arrangement can only reinforce the stability of the instrument.

The pendulums of the new series are equipped with a very low-geared movement acting on the drift screw as well as with a slow movement (but less geared down than the preceding) acting on the "period screw". These movements are activated by long transmission shafts which permit them to be operated from a distance. With these devices, there is no longer any interruption of the recording when a correction of drift becomes necessary. The pendulum resumes its position of equilibrium in one or two hours as has been shown by experiments at Sclaigneaux.

Finally the pendulum cradle for transporting the instrument has been entirely changed. We have arrived at an entirely automatic device which prevents the manipulations necessary in Type I and provides complete security for the threads. This new cradle is not yet being built in series and the ORB pendulum 11 shown in Fig. 23 has not yet been provided with it.

Type III

We have now begun the construction of a pendulum exactly similar to Type II whose base will be constituted, however, by a plate of molten silica with the screws and the feet of invar. In that case, it is possible to eliminate the pressure springs of the tetrahedron on the duraluminum panel and the tetrahedron can be welded directly to the quartz base. Obviously, we will need to wait for the results of the experiment in order to see whether it has produced an actual increase of accuracy and stability corresponding to the cost of construction of such an instrument.

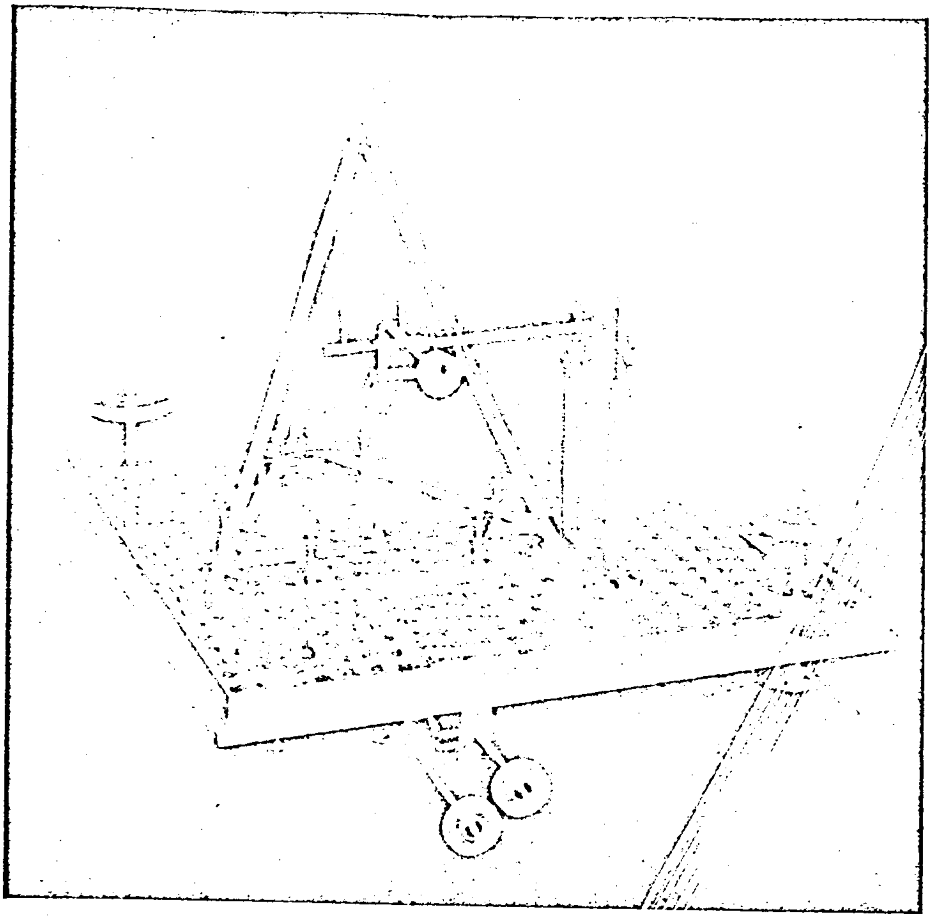


Fig. 23 - ORB pendulum 11 (type II).

The mechanical construction of the prototype and all subsequent instruments was made by M. F. Bourdon, chief mechanic at the Royal Belgian Observatory. The quartz tripod is constructed by the F. Oedenkoven Company at Brussels-Watermael.

There is no need to stress the care required for providing the instruments with the high qualities discussed in the following.

Chapter II

CALIBRATION AND DESIGN OF THE PENDULUM

Calibration of recording units

Accurate calibration of the recordings is of primary importance because it obviously conditions any quantitative interpretation of the observations in regard to the amplitude of the phenomenon. The accuracy of phase is a function of accurate time-keeping of the clock which controls (cf. chapter III) the recording of the parallel and equidistant time markings. A general accuracy of time-keeping within 30 sec is acceptable. For the ORB recorders, the paper speed is 6 mm/hr so that 30 sec correspond to a separating power of 0.05 mm at the instant of the time markings which is evidently sufficient by reason of the presence of small stray oscillations observed on the recordings where 1/10 mm corresponds to about 0.0002". Stricter requirements would be of no significance at the present time except perhaps for certain very special investigations.

Calibration of sensitivity is a problem that can be solved only in two stages:

(a) a base of comparison must be established;

(b) the instrument must be calibrated on this basis.

For inclinometers or horizontal pendulums the basis of comparison may be constituted by a platform supporting the instrument to be calibrated and to which it is possible to impart strictly known micro-inclinations.

Determination of sensitivity of horizontal pendulums

Measurement of sensitivity is a rather complex problem which it is not possible to solve with precision except by means based indirectly on interferometric methods as we shall see further on.

We have already seen that the sensitivity of a horizontal pendulum is determined by the sum $(i + \varepsilon)$ of two small angles. The angle i measures the inclination of the imaginary axis of the pendulum in relation to the vertical and the angle ε depends on the nature and suspension of the pendulum arm.

Let us refer to the expressions

(a) $$(i + \varepsilon) = \frac{T_o^2}{T^2}$$

in which T_o = period of oscillation, generally poorly defined, of the pendulum assumed as suspended vertically, and T - effective period of the pendulum in its normal station.

(b) $$\rho = \frac{d}{i + \varepsilon}$$

in which ρ = angle in radians around which the pendulum arm rotates due to the appearance, in a plane vertical to the pendulum arm, of a component d of deviation from the vertical. The expression $1/i + \varepsilon$ thus corresponds to the coefficient of amplification of the pendulum itself.

At the present time, formula (a) is utilized only as an extrapolation formula making it possible to determine the sensitivity, for a given period of oscillation T of the pendulum, from the sensitivity assumed as known for a period T' different from T.

We then have

$$\frac{(i + \varepsilon)}{(i' + \varepsilon)} = \frac{T'^2}{T^2}$$

Use of formula (a) for measuring sensitivity corresponds to an earlier method which is now obsolete as was pointed out above. We find in fact that this method cannot be used if we desire any degree of precision. It is not possible to suspend the pendulum in a vertical position and to simultaneously conserve, in relation to the pendulum arm, an axis of rotation strictly coinciding with the one which served in the horizontal position. In addition, we would have to take into account the torsional effect of the threads through theoretical calculation. For pendulums suspended by silica threads it would moreover be necessary to break the threads before being able to achieve a vertical suspension.

Some researchers have utilized an inclined platform on which it was possible to operate very accurately and whose variations of inclination could be measured with high accuracy either by means of a high-precision level or a micrometric screw. This method corresponds to an application of formula (b). Placed on a platform, the pendulum is subjected to a variation d of inclination relative to the vertical. The Poggendorf method makes it possible to measure ρ with high precision and the value of $i + \varepsilon$ is derived from formula (b).

By means of a level attached to the platform or by means of a micrometric screw, it is, however, possible accurately to measure only variations d of the inclination with an order of magnitude of 1". Calibration is then made on a base whose intentional variations of inclination are much larger than those which the pendulum is intended to measure on the average. It is therefore necessary, during calibration operations, to diminish the sensitivity of the pendulum by adjusting it to a rather short period of oscillation.

Subsequently, the procedure requires an always somewhat doubtful extrapolation. In order to make this extrapolation, we utilize formula (a) in order to calculate the sensitivity corresponding to a normal period T larger than the period utilized during calibration.

For example, at the start of the practical investigations on earth tides carried out at the Observatory of Uccle, the very accurate Pessler level of the large meridian circle was employed. Of a quality which is exceptional because such levels are rare even in observatories, this level was placed on a yoke provided in the frame of the horizontal pendulum (cf. Fig. 20) and this made it possible, for a given period, to measure the displacements of the luminous spot at a distance of 2 m from the focal point by giving the frame small inclinations by means of the drift screw and then measuring these inclinations as accurately as possible by reading the position of the bubble in the level. We shall see further on that the results were accurate. However, the method does have defects since it consumes much time and effort and requires considerable skill and patience from the operator. It is also limited in its use to periods of less then 60 sec. However, the working period of the pendulums is always greater than 60 sec which requires extrapolation by means of formula (a) as already pointed out. Still the experiment showed that such extrapolation is valid as anticipated in theory. Finally, it was possible only to calibrate rather large displacements of the spot in order to obtain sufficient accuracy in reading the level. Under these conditions, we must assume that the sensitivity of the pendulum is independent from the elongation of the pendulum arm which is certainly not the case for all models of horizontal pendulums presently in use throughout the world. In regard to the ORB pendulums, we should point out that later experiments have not shown any divergences on this point (cf. Table 2).

Expansible crapaudine

In order to avoid the difficulties inherent in the use of levels and micrometric screws, one of the authors developed in 1958 the method of the expansible crapaudine(*). Its functional principle is the following: In order to calibrate a horizontal pendulum, we utilize a crapaudine in the form of a thick-walled manometric capsule placed below the drift screw of the instrument (Figs. 24-26). The chamber of the crapaudine contains a fluid

(*) J. Verbaandert. Etalonnage des pendules horizontaux par crapaudine dilatable étudiée interférométriquement. (Third International Symposium on Earth Tides, pp. 81-90, Trieste, July 1959).

of known pressure. Elastic deformation resulting from the increase of internal pressure raises the upper wall of the crapaudine by lifting the end of the drift screw by an accurately known quantity so that the pendulum becomes inclined similar to the action of an artificial earth tide and is subjected to an angular microdisplacement easy to calculate.

It is hardly necessary to point out that the elastic deformations of the crapaudine are previously calibrated with high exactitude. An interferometric method is used for this purpose and is the only method suitable for measuring microdeformations of this order of magnitude(**).

The characteristics of the steel crapaudines presently utilized at the Royal Belgian Observatory are as follows:

Thickness of upper wall.	5 mm
External diameter	67 mm
Total thickness of capsule	16 mm
Height of the 3 adjustable feet	7 mm
Internal diameter	53 mm
Thickness of manometric cavity	0.2mm
Thickness of bottom	5 mm

The upper face has a small truncated hole in its center intended to receive the point of the drift screw of a horizontal pendulum.

A bushing communicates with the cavity and makes it possible to inject fluid under pressure, e.g., mercury, by means of very flexible tubing of small diameter made of plastic which may have a length of several meters. Deformation tests were made up to pressures of more than 10 kg/cm^2. The deformations always were strictly proportional to pressure.

(**) The description of the specially designed interferometer will be found in the article of J. Verbandert, cited above.

A bulge d of the upper wall of the crapaudine causes the pendulum to incline by an angle which can easily be calculated.

In an ORB pendulum, the 3 set-screws of the base occupy the summit of a rectangular triangle as shown in Figs. 20, 22, 23. The dimensions of such a triangle can be easily measured within 1/10 mm with a relative precision greater than 1/2,000 by placing the pendulum on a plane surface covered with a sheet of thin paper and measuring the gap between the minute pricks left by the points of the screws.

In a specific case, we thus measured 273.5 mm as distance between the drift screw V_1 and the screw V_2 located at the summit of the right angle. In that case, it is easy to calculate that a variation of inclination of 1" results in a bulge d equal to 1.33 μ.

The bulge d in the center of the upper wall of the crapaudine is produced by an increase of internal pressure corresponding to a mercury column of 1,500 mm. It is therefore easy to calibrate in degrees of 0.1" by varying the height of the mercury column in steps of 15 cm.

Fig. 24 - The expandable crapaudine 11.

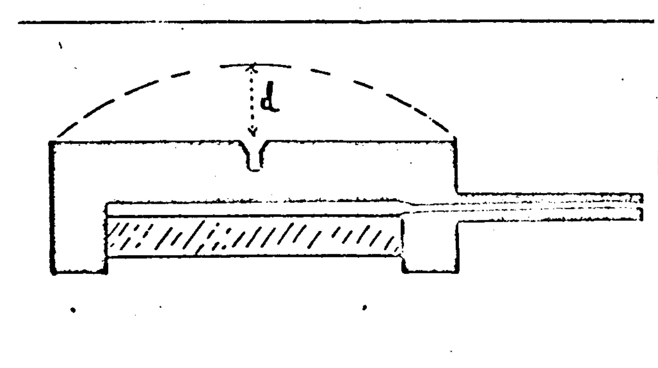


Fig. 25 - Cross sections through center of the expandable crapaudine. The thickness of the internal cavity is greatly exaggerated on the sketch and is actually 0.2 mm. The closure plate is shown in hachures.

Two new applications of the crapaudine

If the end of the flexible tubing containing the mercury connected to the crapaudine is suspended in front of a vertical graduation, we have shown that this produces a bulging of the center of the upper wall strictly proportional to the variations of height shown on the scale.

It is thus possible to investigate dynamically the behavior of the pendulum simply by attaching the end of the tubing at any point of a uniformly rotating vertical disc covering 1 revolution per hour. A sinusoidal wave of known amplitude and period is thus artificially introduced into the movements of the ground on which the instrument rests and which can be measured for its influence on the recording.

It is also possible to utilize the crapaudine for adapting a zero method to the investigation of earth tides. It is sufficient to act, by means of a servomotor, on the level of the mercury which determines the pressure in the crapaudine, in order to constantly maintain, in a slit placed in front of a photoelectric cell, the image of the luminous spot reflected by the mirror of the instrument. The rotation of the servomotor will be controlled in the desired sense by means of the photocell at the instant where, under the influence of a slight movement of the ground, the image tends to leave the slit. In that case, the crapaudine becomes the fundamental element of measurement and compensates by expanding and contracting the natural earth tide and the pendulum itself is simply a zero indicator.

It is obviously necessary to record the variations of height of the mercury which indicate the variations of inclination of the ground.

Some numerical data and technical details on calibration

The elastic characteristics of every crapaudine to be used must be determined experimentally with great care. These characteristics are expressed by a coefficient α of proportionality between the increase ΔH of the internal

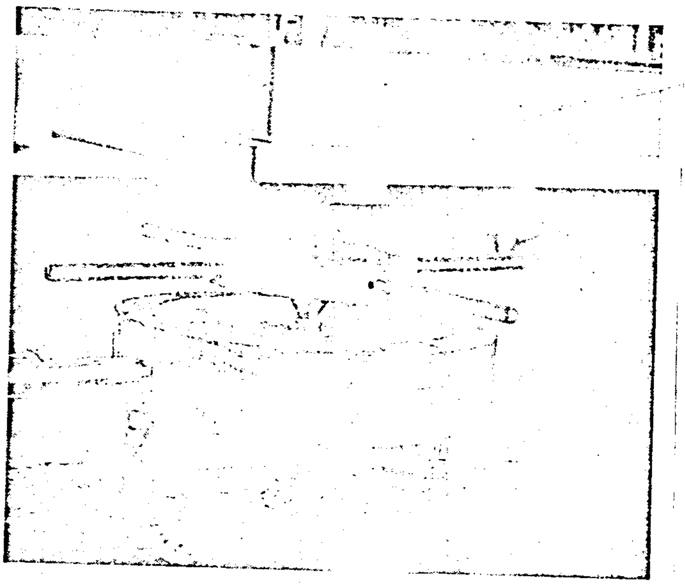


Fig. 26 - Pendulum resting on expandable crapaudine during calibration.

pressure and the amplitude Δd of the expansion of the upper face.

The expression $\Delta d = \alpha \Delta H$ links these three quantities.

Calibration of the crapaudine which comes down in fact to a determination of the coefficient of proportionality α is effected by means of an interferometer based on the green line of mercury with a wavelength $\lambda = 0.546\ \mu$.

The expansion of the crapaudine to be calibrated and placed on the interferometer is the factor which determines the variation of thickness of the sheet of air on whose faces the luminous energy of interference reflects.

In that case, for an expansion of the crapaudine of $0.273\ \mu = \lambda/2$, we observe in the field of vision of the eyepiece of the interferometer the passage from one ring to the next.

For some of the crapaudines utilized at Uccle, we give below the ΔH values corresponding to these observations.

Prototype	$\Delta H = 47$ cm
Crapaudine n° 11	$\Delta H = 37{,}98$ cm
Crapaudine n° 12	$\Delta H = 45{,}18$ cm
Crapaudine n° 13	$\Delta H = 42{,}55$ cm
Crapaudine n° 14	$\Delta H = 40{,}29$ cm

In practice, calibration of the pendulum takes place as follows:

1) Measurement of the distance of the screws of the base as already indicated;

2) Placement of the crapaudine below the drift screw of the pendulum and installation of a horizontal scale graduated in millimeters and located at the place of the slit of the standard recorder. The image of the luminous spot reflected by the mirror of the pendulum is projected on the graduated scale. Subsequently, the distance separating the projection lens and the graduated scale is measured carefully;

3) The period of the pendulum is regulated. We generally utilize four successive periods at around 45, 60, 75, and 90 seconds for which the instrument is calibrated. The period is measured with a chronograph;

4) In strictly determined steps, we vary the height of the mercury controlling pressure in the crapaudine chamber and read the corresponding positions of the image of the luminous spot on the horizontal scale.

It should be noted here that the flexible tubing connected to the crapaudine and at whose end the level of the mercury is noted along a scale of height, is in fact terminated by a small cylindrical reservoir of transparent plastic in which the level of the mercury becomes established.

This is of definite importance for stabilizing the level of the mercury immediately after a change of level has been effected.

Without this precaution and in view of the small cross section of the tubing (2.25 mm^2 internally) and the elasticity of the latter which expands slightly but progressively under the influence of the increase of pressure, the level of the mercury slowly drops during a certain time. With the terminal receiver (internal cross section 10 cm^2), this progressive change of level becomes absolutely negligible.

The height of this mercury container determines the pressure inside the crapaudine. It can be attached by a series of clips placed on a scale and the difference of height of the latter remains unchangeable and is known. Knowing the sensitivity of the crapaudine and the distance between the screws of the instrument, these differences of height can be converted into inclinations of the frame of the pendulum. For example, crapaudine 13 produces, in response to a change of level of 42.55 cm, an increase of elevation of the drift screw of 0.27 micron and therefore an increase of height of 0.006424 micron for 1 cm which gives 0.00484" for a distance of 27.35 cm. Since the clips of the scale are spaced every 25 cm, change of the container from one hook to the next produces a variation of inclination of the pendulum of 0.1210". We then measure on the graduated scale placed in the focus (about 5 m) the displacement in millimeters of the luminous spot reflected by the mirror of the pendulum.

Transfer of the mercury container along the series of hooks finally furnishes the sensitivity of the pendulum (and the dispersion of the measures) for the given period. We repeat the operation for the 4 selected periods and finally make comparison on the basis of the fundamental formula a shown below

$$K = s T^2$$

in which K = fundamental constant of instrument. It is subsequently sufficient to observe the period of oscillation of the installed instrument in order to deduce from this s by calculation.

As an example, we are giving in Table 1 the detailed results obtained for the ORB pendulum 4 which was calibrated both with the aid of a level and of the crapaudine and, to make things quite clear, the sensitivity of the pendulum for the period T = 50 sec and a focal distance of 5 m expressed in seconds of arc per millimeter (s_{50}).

Table 2 shows all details of calibrating the ORB pendulum 11 with the aid of the expansible crapaudine 11. Calibration extended over one week and was made by five different individuals. During the examination of the pendulum, it was decided to measure the period at each hook and, since maximum tolerance of oscillation is only 6 cm, a total of 4 oscillations was measured in order to correspondingly reduce the maximum error.

It will be seen that the mean square error in the constant of the instrument is only 0.68%. It would be even less if we eliminated some measurements made under unfavorable conditions ... [sentence is incomplete in source].

Table 1

Calibration of ORB pendulum 4

Period in sec	Method	K	S_{50}
18,95	N	5,78496	0"00231
28,20	N	5,85328	234
28,36	N	5,71673	229
35,40	N	5,94236	238
44,98	N	5,88306	235
60,28	N	5,71723	229
42,00	C	5,95416	238
90,03	C	5,62083	225
(138,85	C	4,92352 ?	197 ?)

(N = bubble level; C = crapaudine).

with the level $K = 5.81375$ $S_{50} = 232.6$

with the crapaudine $K = 5.78570$ $S_{50} = 231.5$

or a difference of 0.47 %, for this pendulum we choose:

$K = 5.8006$ $S_{50} = 232.05$

Table 2

ORB pendulum 11 Extract from calibration log

hooks	K	T	K	T	K	T	K	T	K	T	K	T
												66,98
1 - 2			6,076		6,464	61	6,311	64,3	5,617		6,404	66,65
2 - 3	6,120		5,936		6,102	62	6,183	64,35	5,391		6,215	66,37
3 - 4	6,174		5,804		6,164	62,3	6,452	63,35	5,852		6,083	65,86
4 - 5	6,220		5,901		6,051	64,7	6,219	64,46	6,538		5,999	66,39
5 - 6	6,254		5,796		5,591	62,3	6,147	60,45	6,586		5,815	65,52
mean	6,1920	50,72	5,9025	62,41	6,0742	62,46	6,2622	63,38	5,9966	65,91	6,1029	66,29
		72,94				78,49				90,37		
1 - 2	6,071	72,74	5,686		6,398	77,39		84,88	6,219	95,16		
2 - 3	5,859	71,13	5,729		6,030	77,95	5,826	82,61	5,448	93,24		
3 - 4	6,304	70,54	5,740		5,958	78,56	5,718	83,62		90,60		
4 - 5	5,771	72,31	6,058		5,849	76,65	6,152	83,91				
5 - 6	5,916	71,79	6,655		5,891	76,88	5,338					
mean	5,9839	71,94	5,9737	73,38	6,0252	77,65	5,7587	83,75	5,8336	92,34		

T = period in sec of time; focal distance = 504 cm.

<u>Table 3</u>

Table of calibration of ORB pendulums

Pendulum number	Method	K	S_{30}
1	N	6,3626	254,50
1*bis* (*)	N	7,0655	282,62
2	N	5,4103	216,41
3	N	5,5955	223,82
4	N + C	5,8006	232,05
5	N	5,9945	239,78
6	N	6,3345	253,51
7	C	6,8060	272,24
8	C	6,1350	245,40
9	C	5,7700	230,80
11	C	6,0577	242,31
12	C	6,2316	249,26
14	C	6,2421	249,68
15	C	5,9121	236,48

(N = bubble level; C = crapaudine).

(*) pendulum 1 after changing quartz threads.

Chapter III

UNDERGROUND STATIONS AT SCLAIGNEAUX AND WARMIFONTAINE

The "recesses"

It is obvious that the conditions of installing the pendulum have quite as much importance as the quality of the instrument itself for the value of the results to be obtained. The instrument should be supported on bedrock with a minimum of intermediate substances; the rock itself must be sufficiently homogeneous and the temperature of the location of the instrument must be absolutely constant. In order to make it thoroughly understood how delicate the problem of the installation of the pendulum is, let us merely point out that if a difference of temperature of 0.003° C became established accidentally between the drift screw and the other two set-screws of the instrument, this would produce a stray deviation of about 5 % of the maximum amplitude of the phenomenon to be measured.

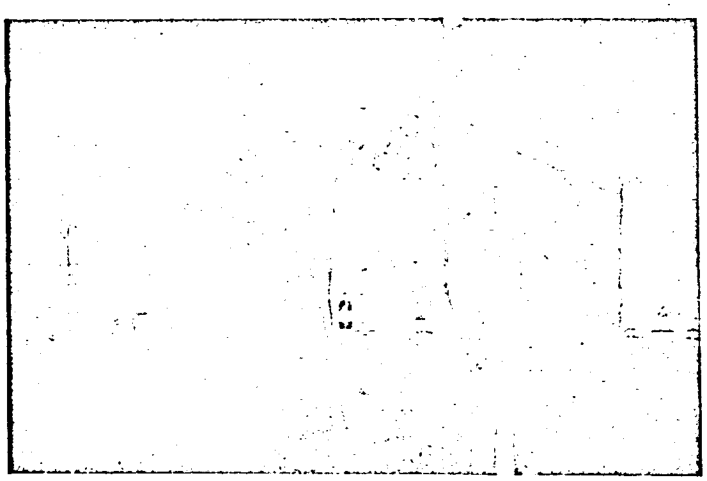


Fig. 27 - Three of the recesses cut into the slate rock at Warmifontaine.

This observation should be sufficient justification for all the precautions taken during installation.

On the other hand, if we refer to the results of calibration of our pendulums given in the preceding chapter, we find that, for a working period of

60-70 sec and a focal distance of 5 m, we find in practice on the recording tape that 1 mm = 0.0015" which corresponds to a change of level of only 20 Ångströms (2/10,000,000 mm)! Now then, we shall show further on that a precision on the order of 0.5 mm is actually attained in reading our recordings, i.e., that we detect variations of level of 10 Angstrom or on the order of magnitude of the dimensions of the unit mesh of quartz (cf. Fig. 10).

Such a result was attained only by a new method of installation. Moreover, other experiments (Tomaschek, Jobert) have also shown the inadvisability of employing artificial supports for the installation of such equipment. Concrete and cement in particular should not be used by reason of instability, creep and sensitivity to moisture.

We selected rock walls in zones of their greatest homogeneity and hardness where recesses for the two pendulums were cut out of the rock without the use of explosives (Fig. 27). They are 50 cm wide by 50 cm deep by 70 cm high so that it is possible to remove the top of the instrument.

By proceeding thus without the use of explosive means, we prevented any fissuring of the rock which would have been deleterious for the stability of the installation. In addition, we thereby achieved so-to-speak an incrustation of the instruments in the rock wall which largely eliminates the sometimes disastrous influence of air currents likely to produce thermal exchange.

All surfaces of the recesses are completely sealed by a coat of "tricosal" covered by a coat of Warrocell, a thermoplastic resin (Fig. 31).

Installation of crapaudines (Fig. 28-32)

The junction between pendulum and rock has been provided by sealing into the rock three stainless steel cylinders (diameter 3 cm, height 5 cm) which rest in three accurately fitted holes drilled in the rock (Fig. 28). The junction between steel and dolomite at Sclaigneaux and between steel and slate at

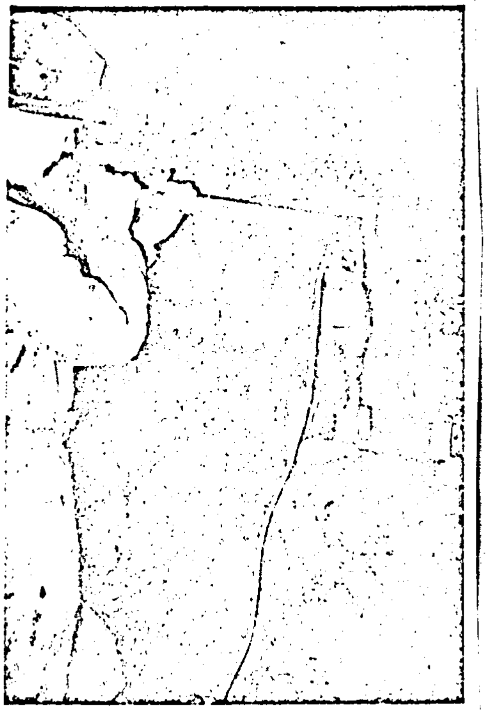


Fig. 28 - Drilling calibrated holes in the rock (Warmifontaine).

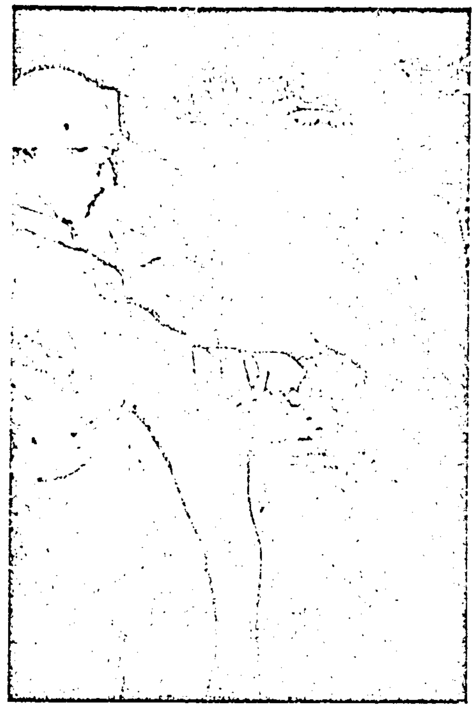


Fig. 29 - Drying the recess with the blow torch (Warmifontaine).

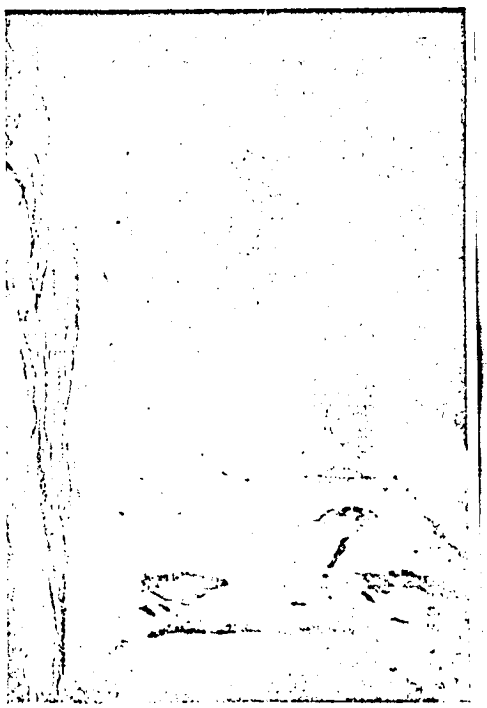


Fig. 30 - The 3 crapaudine cylinders are fastened in the holes. The very dry zone of the recess can be seen from its clear color (at Warmifontaine).

Fig. 31 - Recess coated with Warrocell from which project the upper surfaces of the 3 crapaudine cylinders (Sclaigneaux).

Warmifontaine is provided by a thin coat of a phenol derivative with an additive of silica (thermo-setting resin) which is a strong adhesive and does not shrink. The thickness of the adhesive coating covering the sides of the steel cylinders is about 0.5 mm whereas the lower part of the steel cylinder rests almost directly on the rock, shaped to receive the slightly conical end of the cylinder and separated from it only by a thin coat of phenol resin not exceeding 1/10 mm.

In addition, the vertical surface of the cylinders is threaded to provide optimum adhesion for the resin whereas the bottom is machined in accordance with the conical profile of the drill utilized so as to completely fill the hole. The niche and specifically the drill holes are previously dried out with the torch (Fig. 29) and the resin is polymerized by infrared heat which provides exceptional adhesion within about one hour. Upon completion of the latter process, no further change can take place in the bond of steel and dolomite. The flat upper surface of the cylinder which projects a few millimeters above the floor of the recess (Fig. 30) plays the role of the standard crapaudines distributed as follows in relation to the purpose of the 3 screws:

hole: drift screw

slot: "fixed" point (right angle)

flat: period screw.

To protect the points of the set screws resting on the steel of the crapaudines against any corrosion, the top of the steel cylinders has been machined in the shape of cups which are filled with octyl sebacate, an inert liquid with practically zero vapor tension and an anticorrosive which does not congeal and does not form any acids in the course of time (Fig. 32).

The layer of Warrocell coating the recess is directly joined to these cups in such manner that there is no break in the moisture proofing of the walls and specifically of the stainless-steel resin-coated cylinder-crapaudines and where

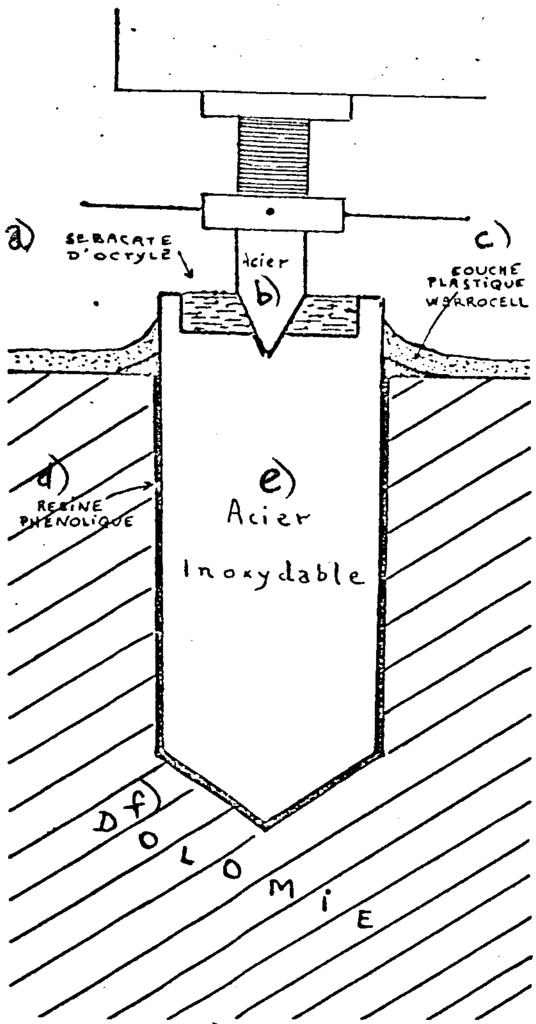


Fig. 32 - **Diagram of the connecting system "horizontal pendulum-terrestrial crust" through crapaudine cylinders grouted in the rock.**

Legend: a = octyl or octyl sebacate; b = steel; c = Warrocell plastic layer; d = phenol resin; e = stainless steel; f = dolomite.

the only free surface, the one supporting the screws, remains submerged under the bath of octyl sebacate.

With the pendulum placed on the three supports thus anchored to the crust of the earth, the recess is sealed hermetically by insulating panels of polystyrene foam with a thickness of 3 cm. The transparent window is constituted by a thick plate with parallel faces which permits passage of the luminous spots. An abundant quantity of a dehydrating agent is also placed in the recess.

The station at Sclaigneaux (Province of Namur)

This underground geophysical station was put into service at the end of September 1959 by the Royal Belgian Observatory in an abandoned mine gallery in a massif of dolomite(*) at Sclaigneaux near Namur.

A long and narrow gallery (average section 1.60 x 1.60 m) cuts horizontally, at the level of the Meuse River, through the base of a massif of "viseenne" dolomite at the northern base of the synclinorium of Namur south of the large Landen fault and east of the sigmoid of Gelbresse (Figs. 35 and 36) and has the following coordinates:

astronomic latitude $\varphi = 50^\circ 29' 35''$ N

longitude $\lambda = 5^\circ\ 1'\ 15''$ E

geocentric latitude $\psi = 50^\circ 18' 15''$ N

ground level altitude : 170 m.

The instruments are installed at a depth of 85 m below ground level. The gallery was cut around 1850 at a time where high explosives were not yet available and when exclusively manual labor and black powder were used; we may assume that it suffered no detrimental concussion. In addition, the small cross section of the gallery and the absence of large diggings in the vicinity of the instruments constitute a certain guarantee.

The gallery is obstructed 800 m further on and is said to continue for another 1,000 m to a point where formerly iron ore was brought to the surface. What is important is that the dolomite was not extracted from the gallery.

The location at Sclaigneaux was indicated to us by the geologist A. Wéry who as such has participated in all research undertaken in Belgium for the investigation of the earth tides. It evidently corresponds to all the criteria established by this author in his memorandum "Sites of Underground Geophysical

(*) Dolomite is a calcium/magnesium carbonate (54.3 % CO_2Ca, 45.7 % CO_2Mg) with a density of 2.8-2.9.

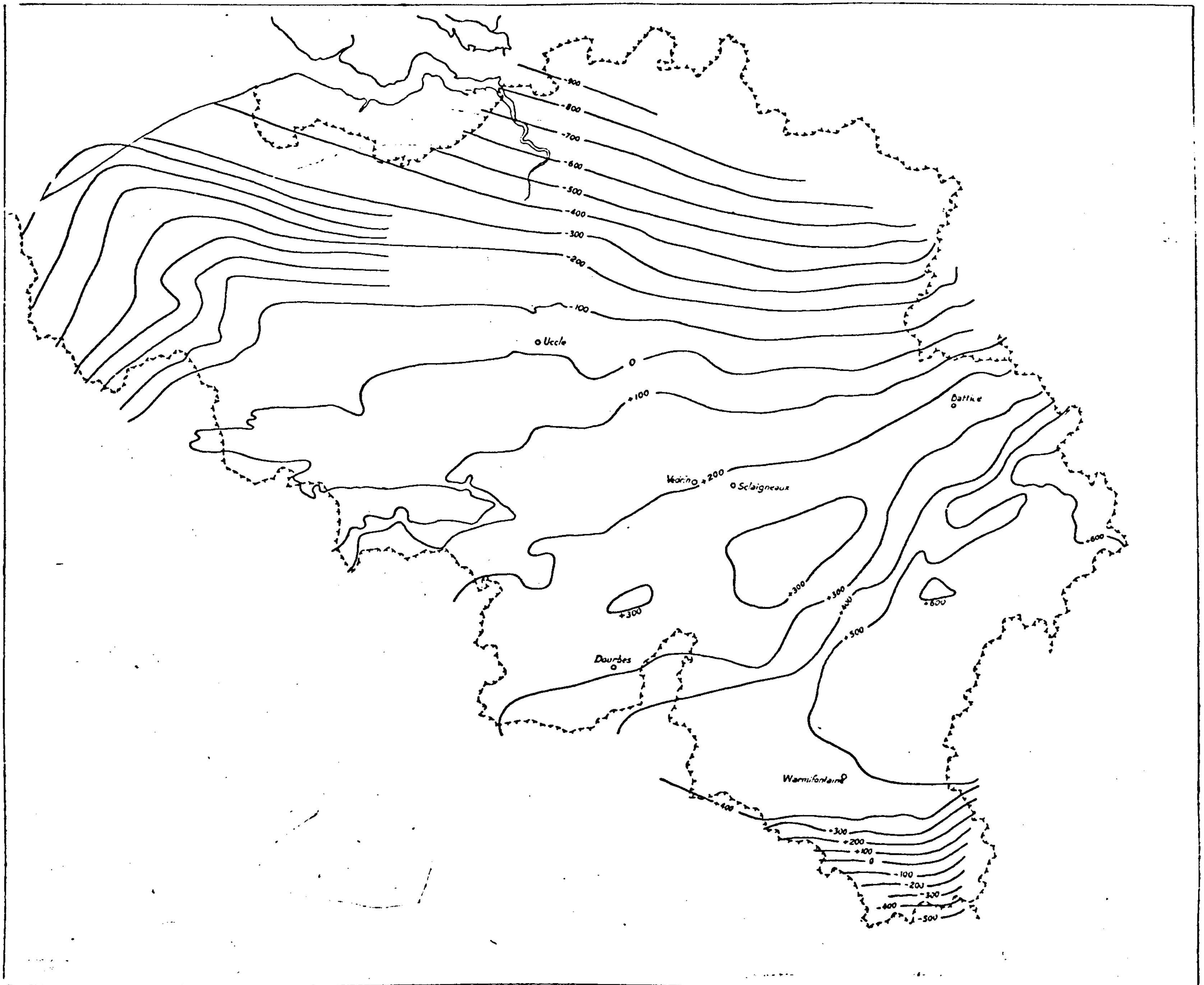


Fig. 33 - Location of the Belgian earth-tide stations in relation to the paleozoic bedrock for which contour lines are given.

Stations of the Royal Belgian Observatory" (Ciel et Terre, No. 11-12, 1960).

The soundest zones of the rock were selected with great care. We selected two points where the arrangement of the gallery made it possible to provide optimum orientation north-south and east-west. In May 1959, the first two recesses for pendulums were cut out of the rock here.

Cutting the rock by hand was difficult due to the hardness of the dolomite and required almost one week by two men alternating in the work.

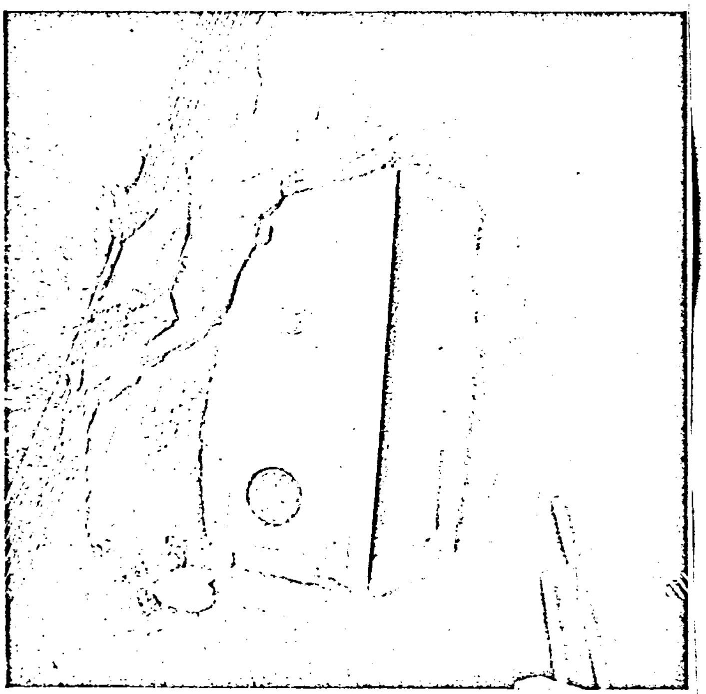
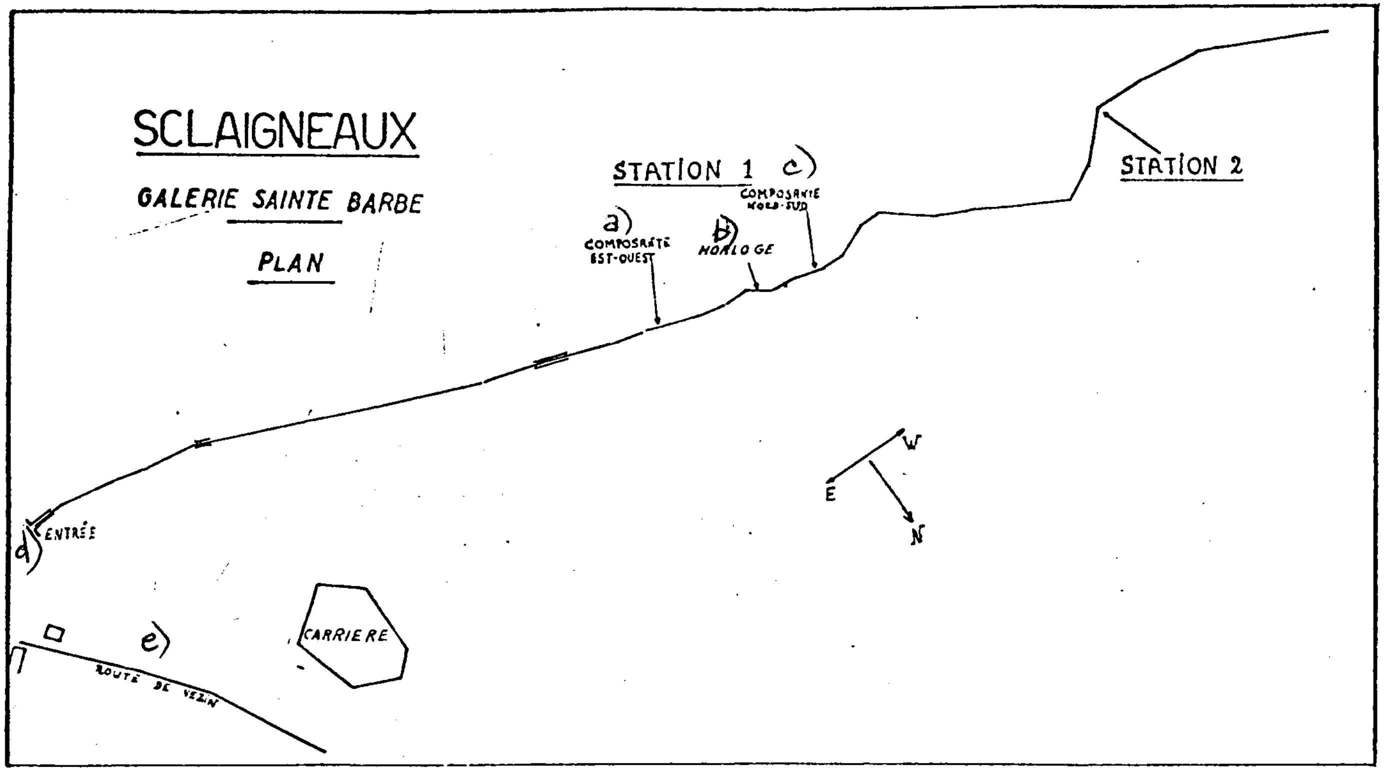


Fig. 34 - Recess of ORB pendulum 1 at Sclaigneaux (E-W component).

The first recess houses the pendulum giving the component east-west (Figs. 34 and 37). It is located 400 m from the entry to the gallery. The second pendulum for the component north-south is located 500 m from the entry and the clock has been placed between these two points.

Construction and equipment of the two recesses was finished in August 1959. The ORB pendulum 4 was placed in service for the north-south component



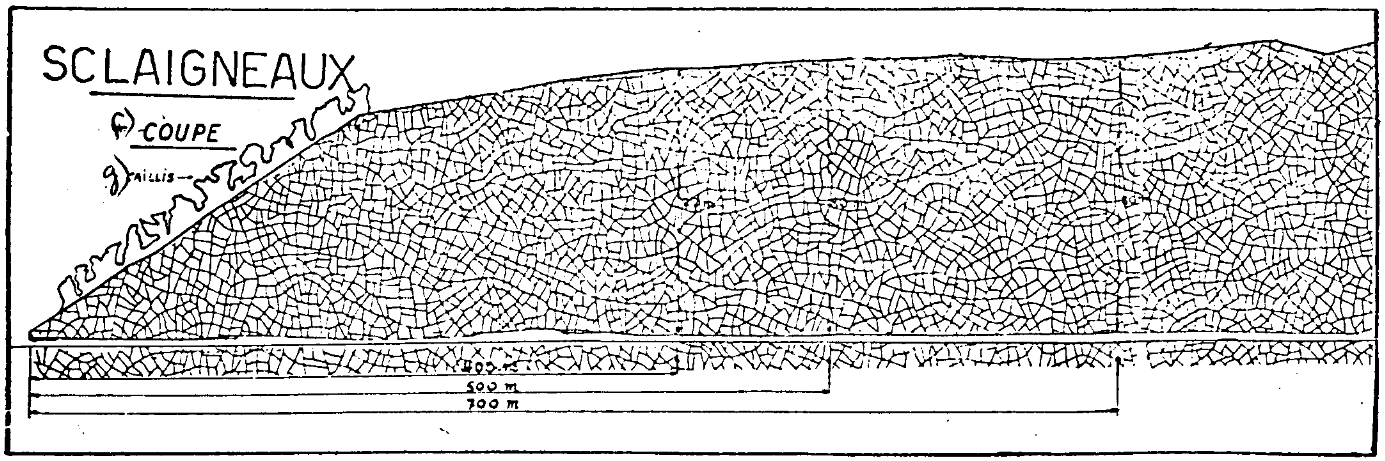


Fig. 35

Legend: a) E-W component; b) clock; c) N-S component; d) entrance; e) highway to Vezin; f) section; g) woods.

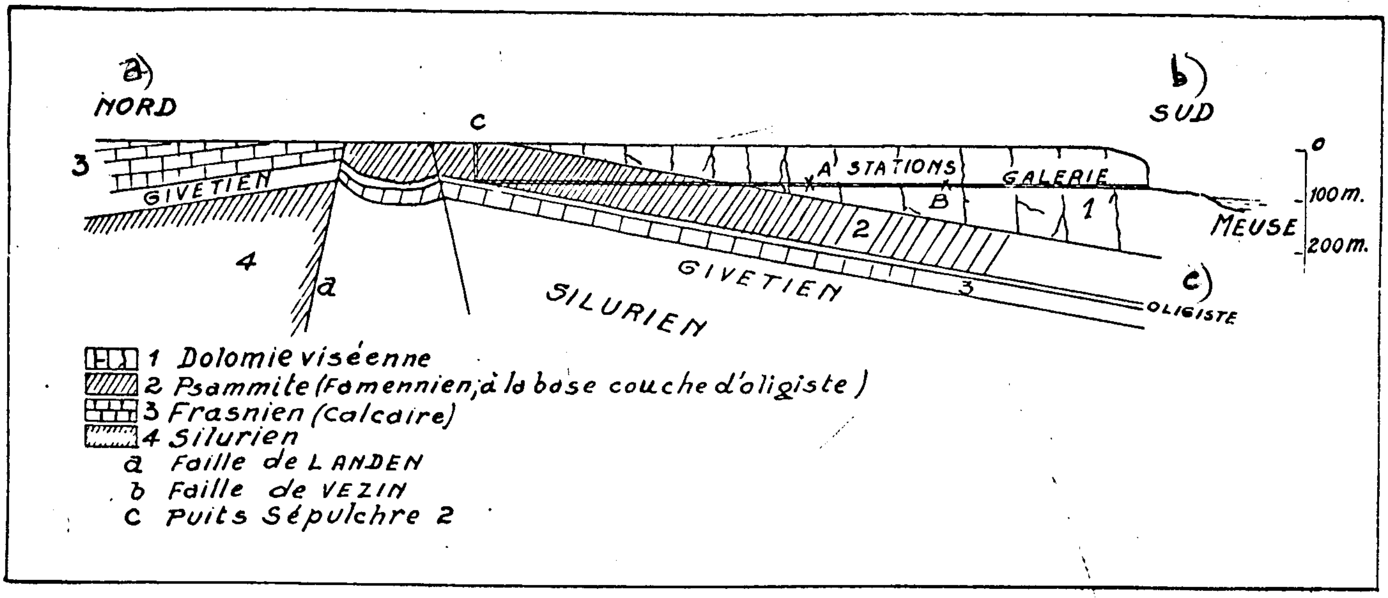


Fig. 36 - Geological cross section for the Sclaigneaux station.

1 = Visean dolomite; 2 = summit Psammite (fomennian with subjacent layer of oligist); 3 = Frasnian (limestone); 4 = Silurian. a = Landenford; b = Vezin fault; c = Sepulchre-2 mine shaft.

Legend: a) north; b) south; c) oligist.

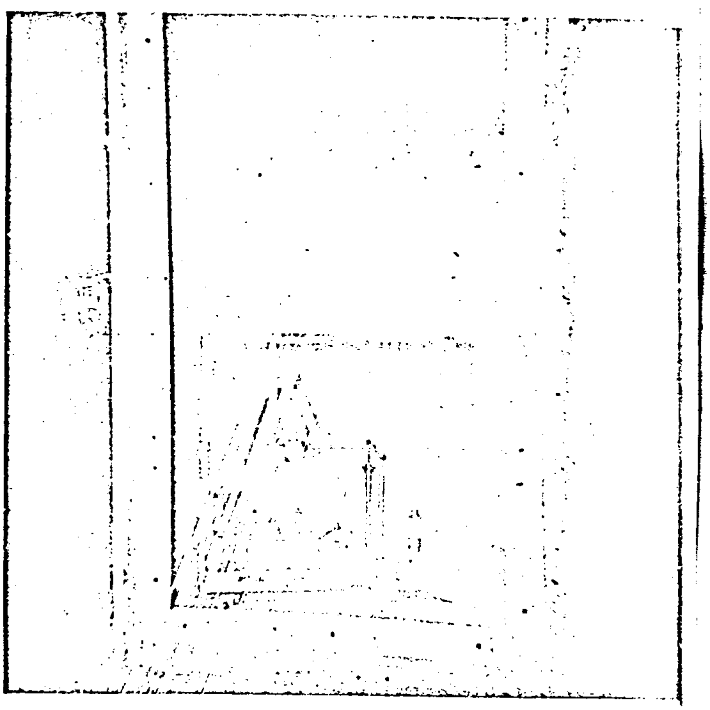


Fig. 37 - ORB pendulum 1 in recess at Sclaigneaux.

between 7 and 14 September by utilizing a small provisional recorder. Under these conditions, a very satisfactory recording was obtained at the moment of the partial eclipse of the sun on 2 October 1959 (Fig. 38).

During an eclipse, the attraction of the moon and of the sun superpose each other perfectly and we see on the photograph that the successive low tides have the same extent. This circumstance usually cannot be checked as is shown by photographs 39 and 40 on which we see immediately the effect of the phases of the moon: In the first and last quarter, the action of the sun is opposed to that of the moon and the amplitude of the variation is minimum. In conjunction (new moon) and in opposition (full moon), on the other hand, the effects are added and amplitude is maximum. However, the difference of the average longitude and declinations of the two celestial bodies is responsible for the fact that two successive low tides are not equal (role of diurnal components of the tide). It is therefore only at the times of eclipse (mean longitudes and equal declinations) that we observe an almost perfect sinusoid like that of 2 October 1959.

Since 20 October 1959, the recordings are made by the large-scale and sealed recorder described further below.

Fig. 46 represents the table of activity of the Belgian stations of earth tides and gives an idea of the work already accomplished at this station. The description of the secondary equipment and the numerical data obtained will be given further below.

At the beginning, several necessary adjustments in the secondary equipment caused interruptions in recording. On 12 July 1960, the ORB pendulum 4 was removed and replaced by the ORB pendulum 9 equipped with a slow drift movement described further back. Since then, recording has been continuous.

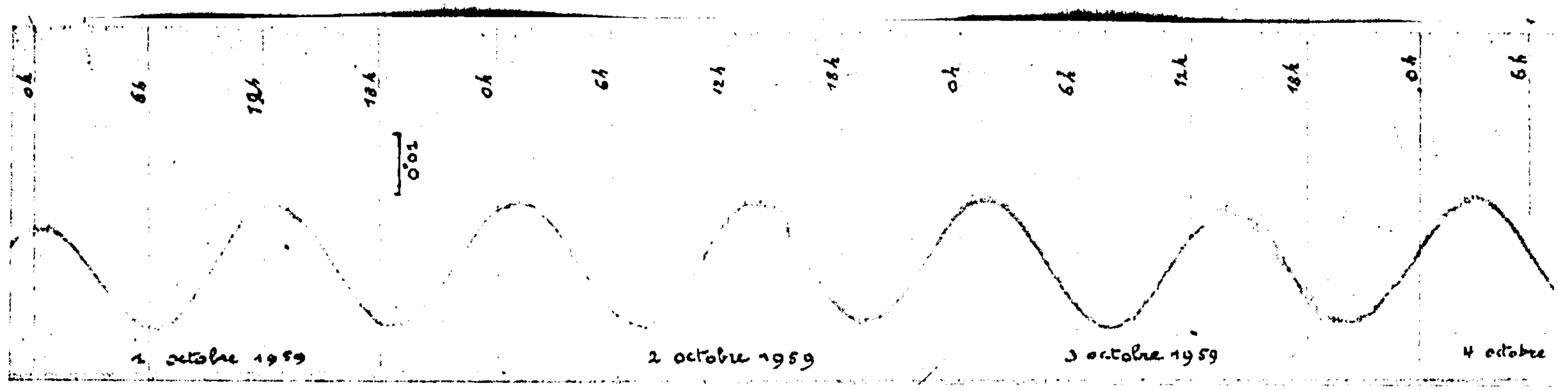


Fig. 38 - Recording of the horizontal N-S component of the earth tide at Sclaigneaux by ORB pendulum 4 during the partial solar eclipse of 2 October 1959 (reproduced in actual size and obtained on a temporary recorder).

In the meantime, observations of the component east-west have been provided since 29 March 1960 by the ORB pendulum 1, our early prototype which had previously been utilized without success at Vedrin(*) and which has been equipped with the slow drift movement of type II pendulums. Recording has here continued since that date except for infrequent interruptions due to minor mechanical trouble in the recorder or the clock.

At the present time, two new recesses have been cut into the rock at a distance of 700 m from the entry for housing two new pendulums (type II) in order to have optimum control of the recordings and to investigate specifically certain short-period micro-inclinations observed in the already installed pendulums.

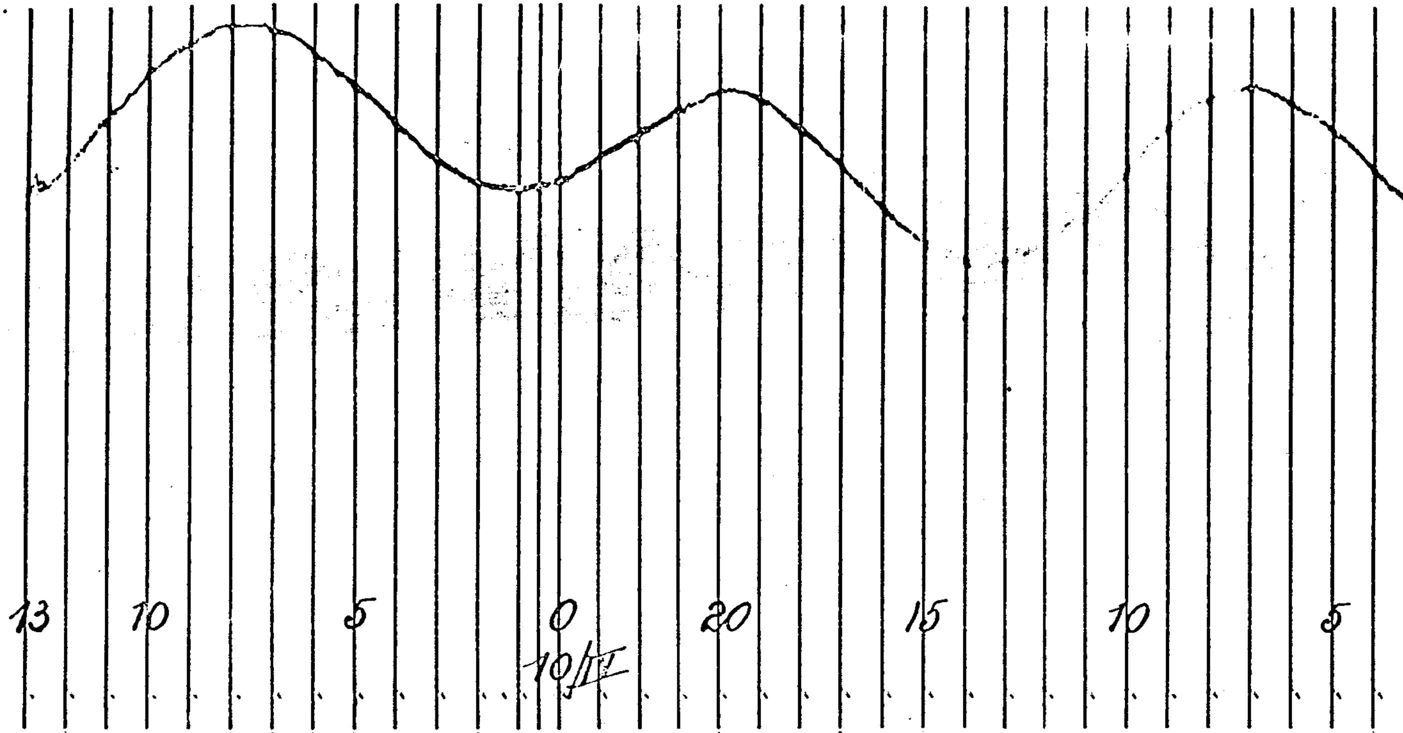
In addition, a third recess has been cut at a point 750 m from the entry where we expect to install the prototype of the type III pendulum. These units were scheduled to begin operating in 1961.

Grateful acknowledgment for their assistance and cooperation is due to A. Wéry, geologist and mining engineer, for his information on operating and closed Belgian mines and his technical guidance; to Messrs. Tonglet, M. Tirtia and A. Vanhorebeck, of the Dolomite Quarry at Sclaigneaux; to Messrs. A. Agie de Selsaete and Sépulchre of the Société Savgaz for their permission to effect the physical installation; and to the Société Isobelec at Sclessin for the installation of the steel bars in the dolomite and the sealing of the recess walls with Warrocell (a phenol derivative).

<u>The Warmifontaine station (Province of Luxembourg)</u>

This underground geophysical station was put into operation at the end of of 1960 by the Royal Belgian Observatory in a gallery in the slate bed of <u>Warmifontaine</u> with the following coordinates:

(*) Preliminary tests in the Pyrite Mine at Vedrin were carried out under the customary elementary conditions of placing the instruments on a concrete base and grouting the standard crapaudines in cement.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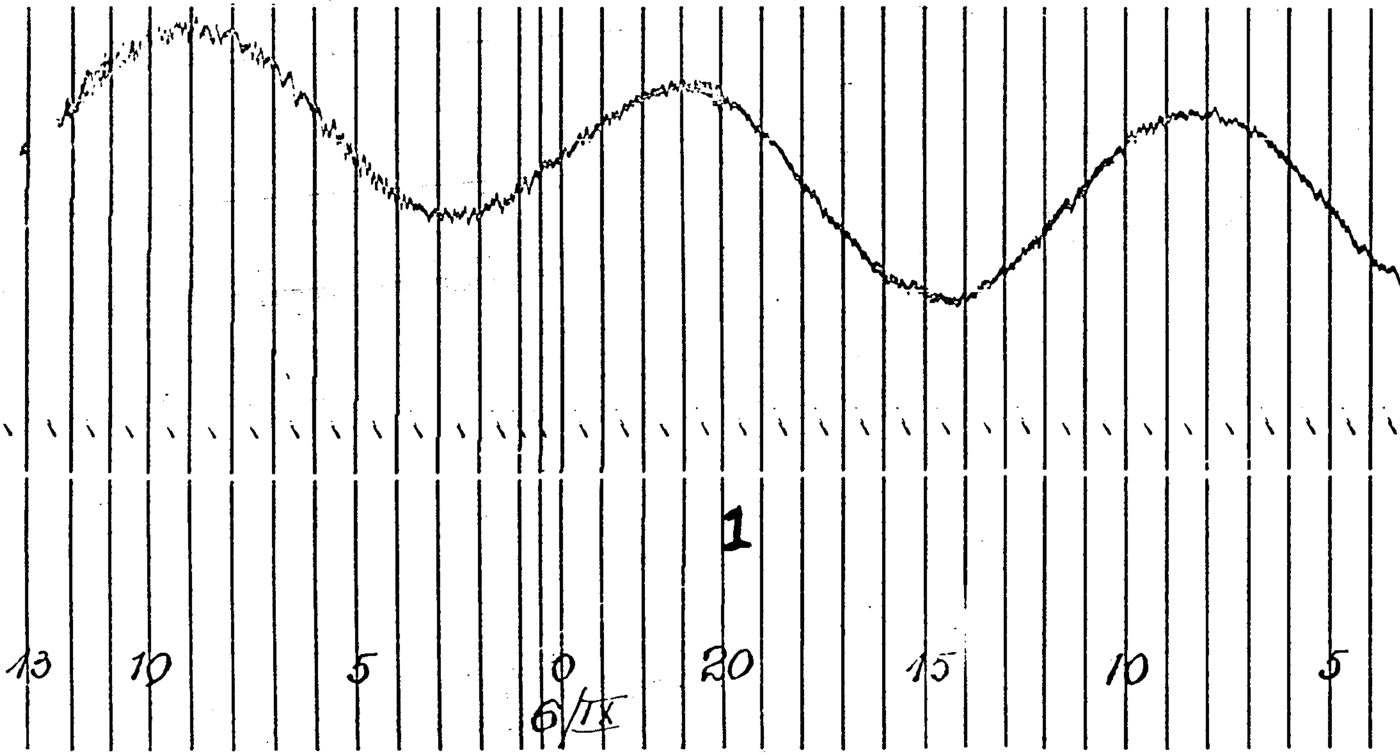
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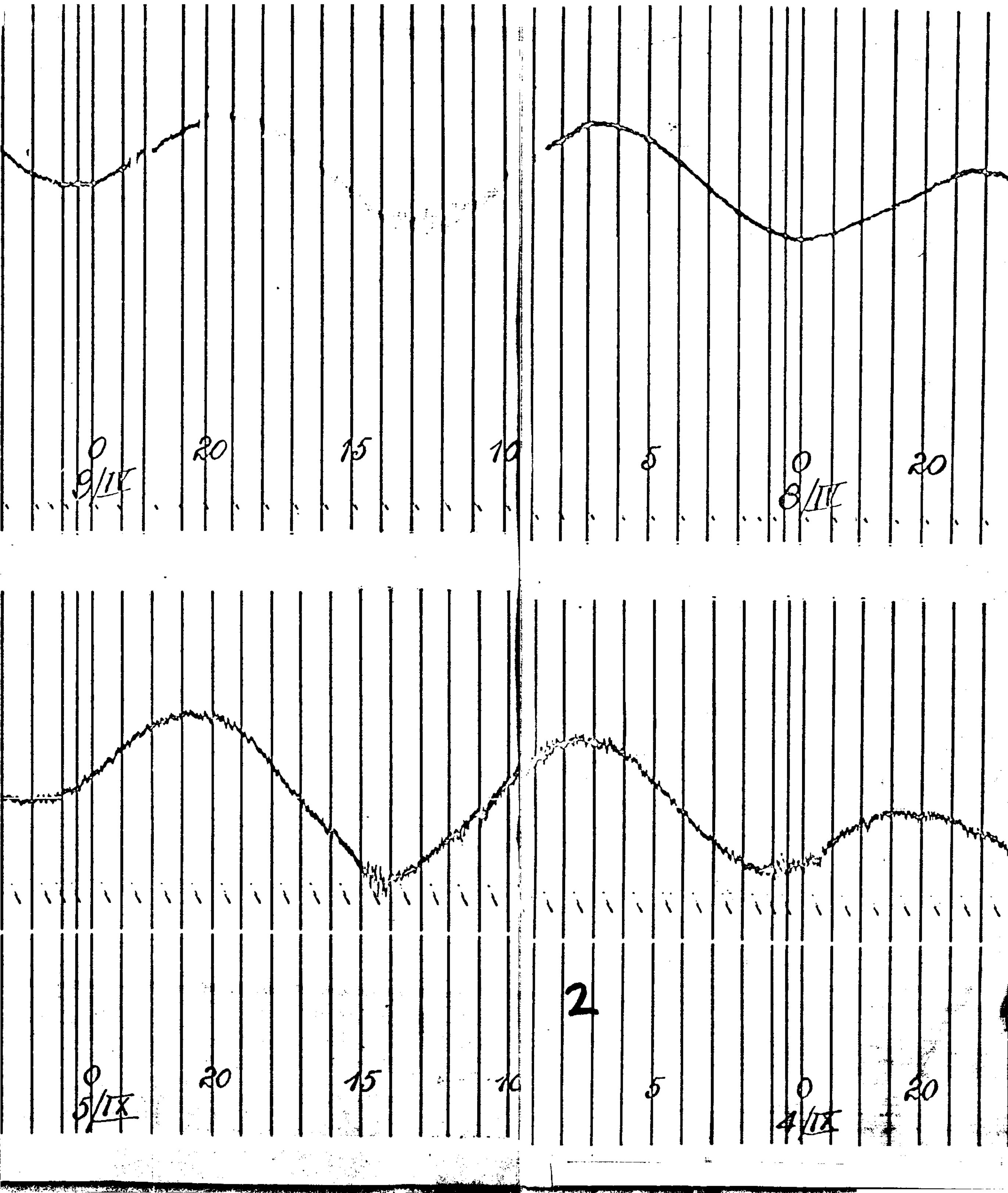

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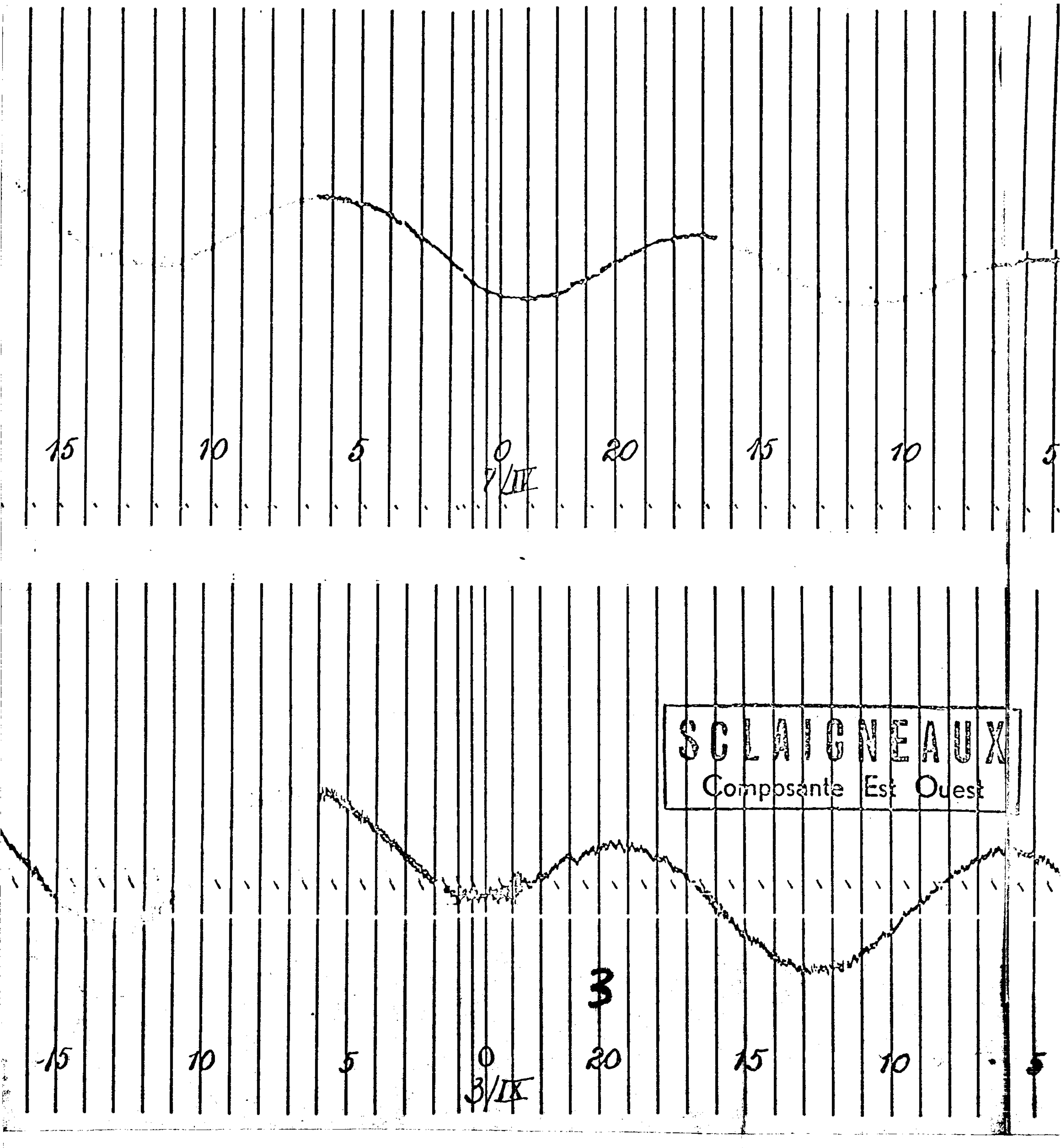
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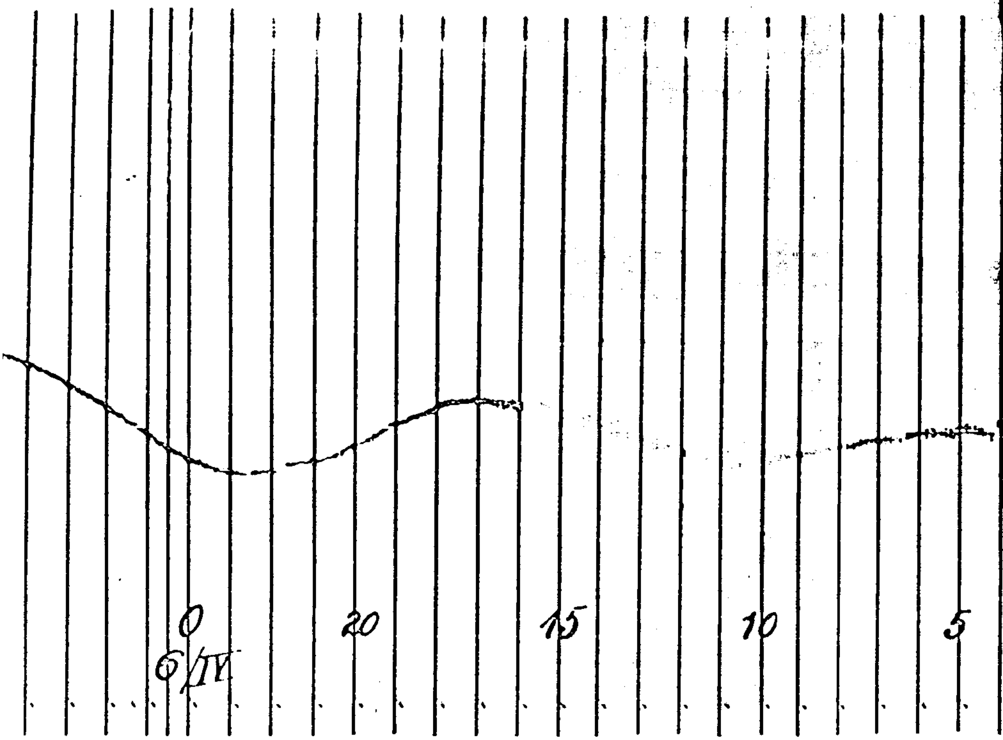
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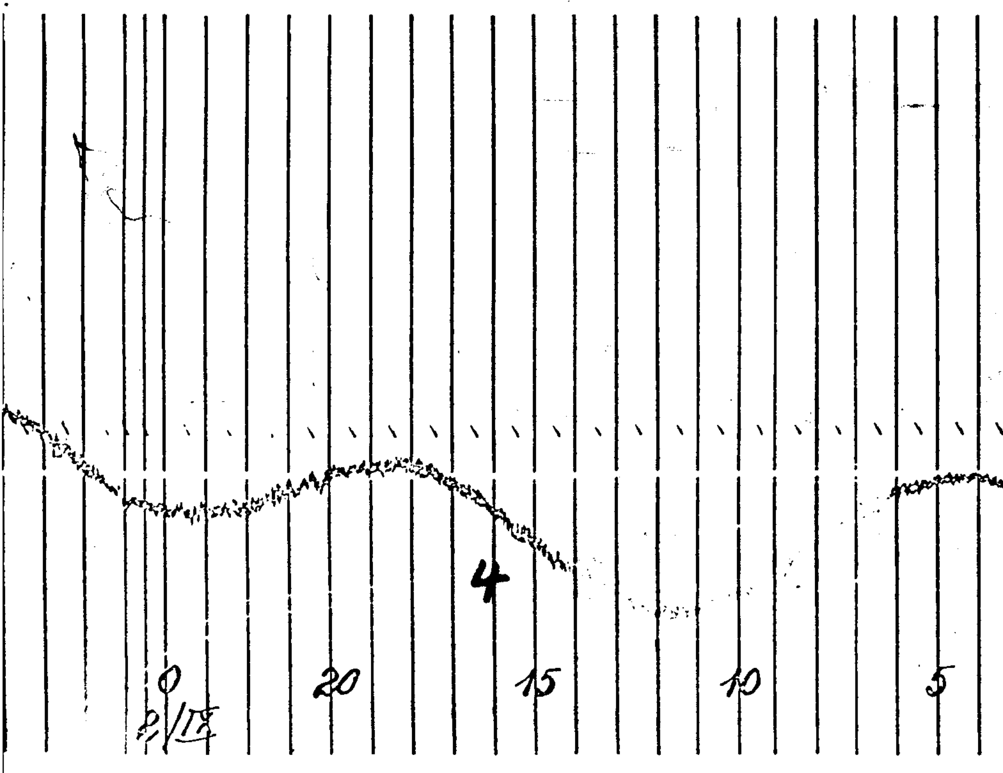
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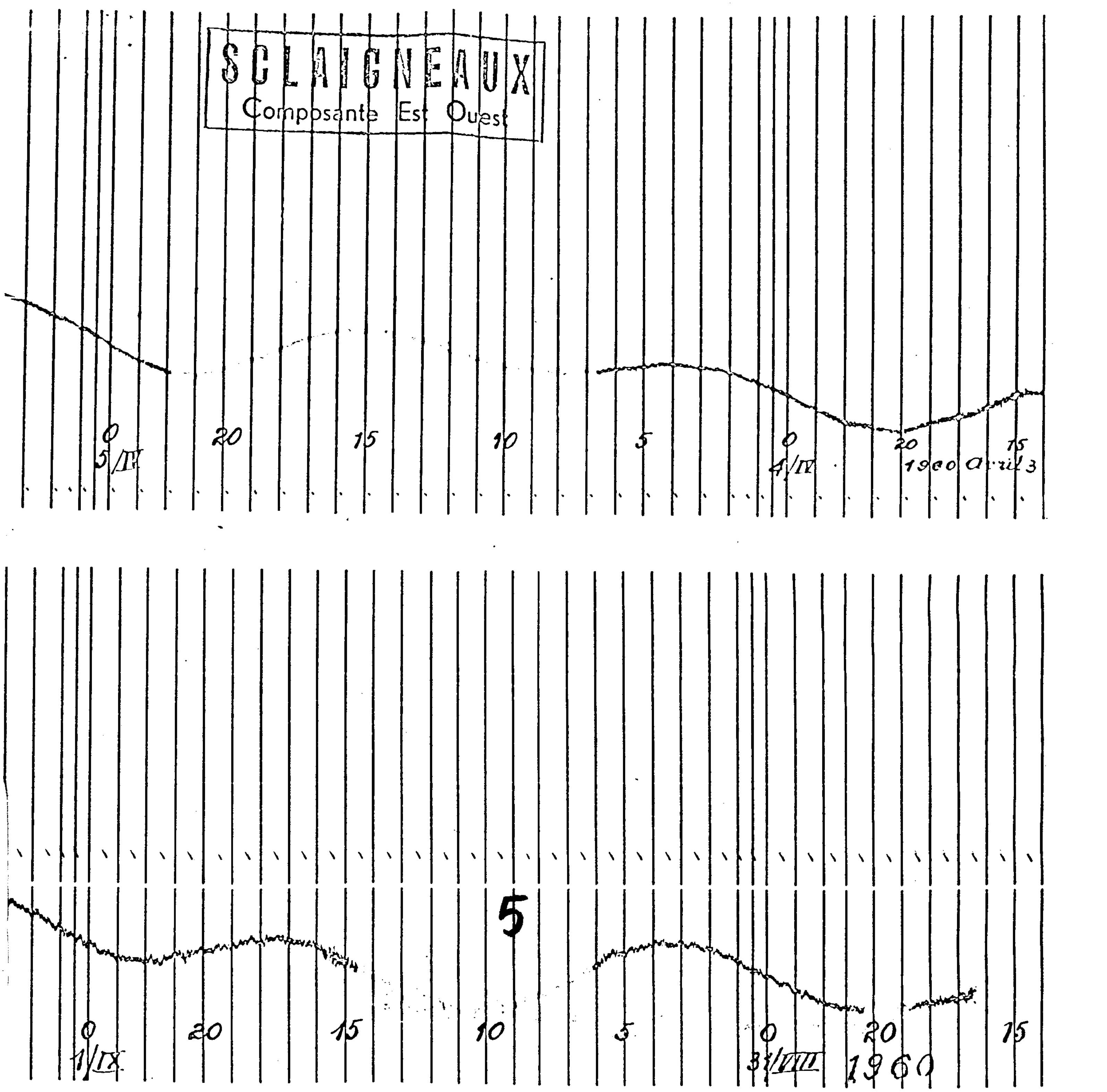


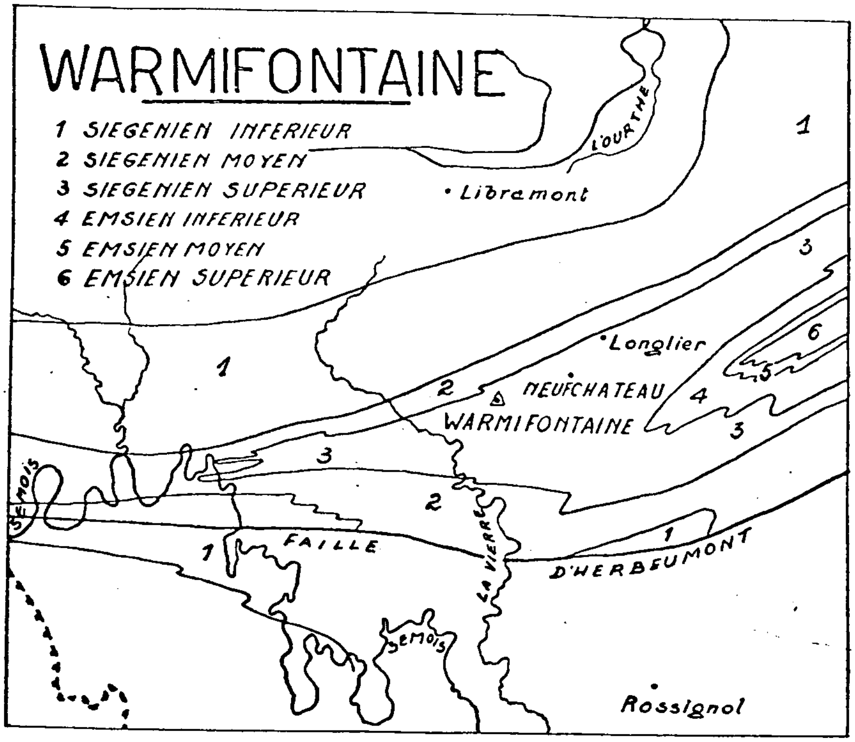
Figs. 39 & 40. Recordings of earth tide (deviations from the vertical, at Sclaigneaux of the E-W component [Composante Est Ouest] (ORB pendulum 1 reproduced in actual size: 1 mm = 0"0016 (the vertical lines are the hour marks).)) [Foldout] astronomical latitude $\varphi = 49^\circ 50'$ N

geocentric latitude $\psi = 49^\circ 39'$ N

longitude $\lambda = 5^\circ 22' 50''$ E

altitude of ground level $h = 390$ m

depth of gallery p = 148 m.



Legend:
Inferieur = Lower
Moyen = Middle
Superieur = Upper
Faille d' = Fault of

Fig. 41

This is a gallery cut into slate consisting of pyllades (slatey schist) of the "siegenian" (upper Hunsrück) of the Devonian basin of the Eifel.

The gallery starts from the mine shaft at a depth of 148 m and has a length of 120 m in the non-exploited zone of the slate bed.

The construction of an experimental dilatometer(*) was undertaken at the far end of the gallery. Some 20 m ahead of the dilatometer have been cut some 3 observation chambers intended for a gravimetric-tide station (G), a seismological

(*) P. MELCHIOR - Construction of experimental dilatometer. (Deuxième Colloque International de la Commission du CSAGI pour l'étude des marées terrestres - Munich 1958). Comm. Obs. Royal Belgique n° 142, Ser. Geoph. 47 - 1958.

station (S) and a clinometric station (C) equipped with ORB horizontal pendulums.

Four recesses were also cut into the walls at this point in which the crapaudine cylinders were installed with the aid of the phenol derivative (Fig. 27-30).

The ORB pendulums 4 and 11 were installed and record respectively the NS- and EW-components. They were given characteristic periods of about 70 secs.

Our selection of this station is also due to M. Wéry, the geological engineer.

For this installation, grateful acknowledgment for the preparatory and practical work is due to the Société des Ardoisières of Warmifontaine, M. Schoemans and M. Lodez; and to M. Frankart for the weekly collection of the recording tapes and necessary corrections of drift.

<u>Some practical pointers on handling the instruments</u>

It seems appropriate to add some suggestions for the handling of the horizontal pendulums during the calibration and installation operations.

It will be superfluous to stress the care with which these operations must be effected and it is obvious that the pendulum must not be handled roughly.

The instrument is transported in a wooden case, suitably held in place by foam rubber pads, and should not be left out of sight at any time during the transport.

Before setting the instrument on the expandable crapaudine and the set screws or on the socket bars of its recess, the surface of these elements is examined to determine whether it is perfectly clean and free of any solid particles such as rock dust.

Unless the operator is already very familiar with handling the instruments, it is suggested to tighten the period screw sufficiently so that the pendulum will have a very short period and consequently very little sensitivity before removing the pads.

At the instant where the locking cradle is lowered and the arm has just been liberated, we must note immediately the direction in which it tends to swing and maintain the arm centered by activating the drift screw while the cradle is fully lowered.

At that moment, the pendulum oscillates rapidly and these oscillations can be dampened by exerting slight pressure with the fingers on the protective casing in phase opposition with the movement of the pendulum.

We then gradually loosen the period screw to raise it gradually toward the working period of the pendulum of about 70 sec.

From 10 to 15 sec after the start, the period increases slightly and the screw is rotated one-fourth of a turn at a time while carefully correcting the azimuth of the arm and damping the oscillations between each operation.

After 40 sec, greater care must be exercised by rotating the drift screw millimeter by millimeter and carefully checking the azimuth and the attenuation.

After about 70 sec, the slow movement of the period and drift screw are set along the half-way position and the collars are firmly tightened. Subsequently all adjustments are made only with the aid of these movements.

Installation of the spot lights is facilitated by solidly assembling "dexion" steel brackets to the supporting column of the recorders.

After the location has been completely prepared, the time required for installation of a pendulum is about one hour.

Calibration in steps of six scale positions lasts about 40 minutes. With each change of position, the induced oscillation is attenuated for some instants by manually raising and lowering the mercury cup in phase opposition to the oscillation of the arm. (One or two movements are often sufficient for a somewhat skilled operator.)

Installation of the ORB pendulums 4 and 11 at Warmifontaine including adjustment of the spot lights, the recorders and of the clock required about 7 hours for two persons.

However, the preparation of the chamber including cutting the recesses, drilling and grouting of the crapaudines, sealing of the recess walls, installation of the recess doors, construction of the supporting columns, electrical connections, and chamber doors extended over several months.

Before all such essential preliminary work has not been terminated, the installation of a pendulum should not be begun.

However, we suggest to place the pendulum in the chamber for several days and, in practice, one week before attempting to adjust it to a proper period and adjust the projectors and recorders.

During this time, the instrument is able to absorb the mean temperature of the location in all its parts.

In order to accomplish this, the instrument may be left in its case on location for a week before being unpacked.

Installation of auxiliary equipment

The horizontal pendulum is the most delicate of all the equipment which constitutes a clinometric station.

However, considering the ideal conditions of installation in a hermetically sealed recess in the rock and protected against any disturbing agent, once the instrument is installed, it can operate for years without any other care than an occasional slight correction of drift effected by means of the geared-down screw of the slow movement (cf. Fig. 23) which extends out of the recess, like that of the period screw, through a small hole drilled in the recess door.

During such operation, the pendulum is subjected to a slight disturbance which provokes oscillations of very small amplitude at the moving arm (a sweep of 3-4 cm on the slot of the recorder).

This circumstance is utilized to measure the period which is the characteristic of adjustment for sensitivity of the instrument. The drift correction is practically the only case where it is necessary to act on the pendulum itself.

By contrast, if we consider the auxiliary equipment and specifically the photographic recorder, it is absolutely necessary to very carefully inspect their operation if we desire to prevent gaps in the recordings whose causes can be manifold. We must not forget that, in the problem of terrestrial tides, a recording has no value unless it covers a minimum of 29 days (lunar month). We can then perform a calculation of harmonic analysis (chapter IV) and, if the recording is for a longer period, we perform successive analyses, staggered from 10 to 10 days and therefore overlapping by 19 days. This makes it possible to examine the evolution of the phenomenon in detail. Fig. 46 shows a synoptic table of calculation already developed on the basis of the recordings of each of the instruments of earth tides in operation in Belgium.

In the mining galleries where such instruments are in permanent operation, exceptional hygrometric conditions prevail as a rule; the air is supersaturated with moisture and water often drips from the ceiling.

Consequently the geophysicist must exercise all possible care so that the installation will satisfy very specific conditions if he does not want to be forced to recognize, after months of discouraging tests, that the observations obtained are not worth final evaluation by reason of the innumerable gaps in recording.

As an example, it happens frequently that the windows closing the recess of the pendulum and the photographic recorder become fogged. However, these windows are intended for the passage of the light beam of the spot lights and, under such conditions, the quality of the photographic recording becomes poor and frequently does not even show on the film if the fog on the windows becomes so thick as to make them practically opaque to light.

Another case may also be found. If the housing of the recorder is not absolutely hermetically sealed, the air on the inside gathers moisture and the very hygroscopic photographic film becomes buckled and deformed under the influence of the water vapor. When this occurs, it is obvious that the precision of the recordings is also inadequate.

In order to prevent numerous difficulties of all kinds, the Royal Belgian Observatory has designed and constructed a prototype of a special recorder which is noteworthy for its insensitivity to moisture together with a multiple spot light adjustable in height, direction and distance.

The prototype of the recorder and the projectors were constructed by Mr. Geerlandt, chief technician at the Observatory. They are now being built in series by Ateliers de Construction Mecanique Vanderstock at Brussels.

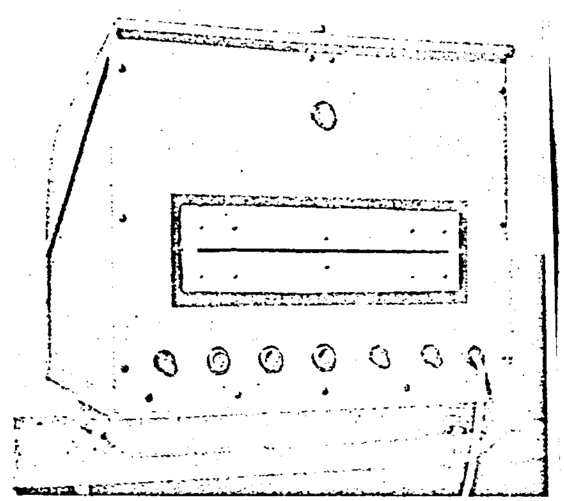


Fig. 42 - Front view of ORB photographic recorder; note white screen showing only the entry slot of the luminous beam.

There exist two types of photographic "spot" recorders: (1) with continuously moving film unwinding from a reel which represents an amount of film sufficient for several weeks; (2) with a rotating drum provided with a sheet

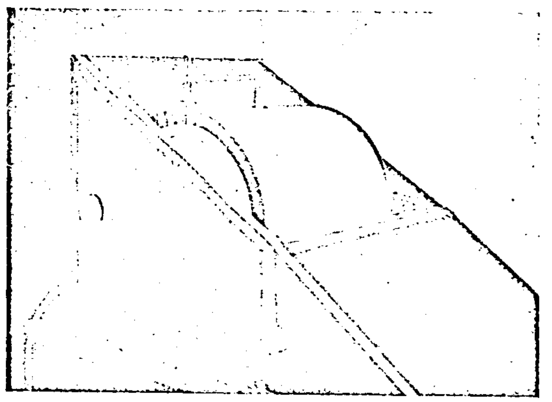


Fig. 43 - ORB recorder opened; note the graduated circle of the drum and the observation window for the circle.

of sensitive paper around its periphery and making one complete revolution per week.

The second type is generally of simpler construction and its operation is more reliable. On the other hand, it requires that the operator visit the station at least once a week, which is necessary in any event in order to be able to examine the installation for some unexpected defect in operation.

Our recorder is a drum recorder, which has the following main characteristics: diameter of drum = 30 cm; width of drum = 30 cm; photographic film whose recording surface is 90 x 30 cm. The drum is driven by an electric motor of the phonic-wheel type with geared-down transmission and makes one revolution per week. This corresponds to an hourly film speed of 6 mm across the slot of the recorder.

The drum and all auxiliary components of the recorder are enclosed in a metal case which is sealed as hermetically as possible. In Fig. 43, it will be noted that the cover which fits approximately around the drum and opens diagonally so that the drum is completely disengaged for replacing the photographic paper itself which is attached to the drum without any break in continuity simply by using adhesive tape which is coated on both sides.

The casing is sealed hermetically by "plastic foam" gaskets which are compressed when the cover is closed.

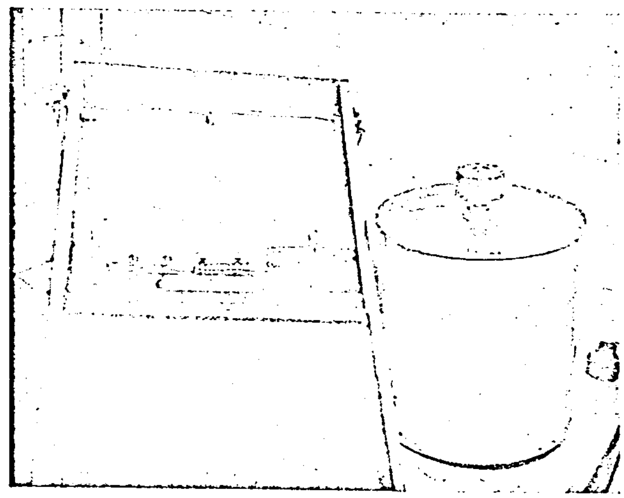


Fig. 44 - ORB recorder disassembled; note the electric heater in the bottom of the housing; notice on right drum motor on drum shaft and the guide rims for the paper.

The drum consists entirely of bakelite with heavy rims accurately centered on the lathe and fitting into a peripheral groove in the interior of the hollow cylinder while extending slightly along the periphery of the cylinder in such manner as to provide accurate guidance in attaching the photographic film (cf. Fig. 44). An accurately centered heavy metal shaft runs through the drum. The shaft ends run in roller bearings which are supported by two vertical uprights. A geared-down transmission provides uniform rotation of the drum. During adjustment, the latter has been dynamically balanced so that it runs perfectly true. Motive power is provided by a synchronous motor of the phonic-wheel type operating on net work current 220 V, 50 c/s. The motor is of the same type and of the same order of magnitude as the motors of the clocks which operate directly on network AC lighting current, and consumes 2 W. The revolutions of the phonic-wheel are stepped down to one rotation per week by a system of gears of very small dimension completely enclosed in the small cylindrical casing which houses the motor, (transmission ratio = $2 \times 10^6 : 1$). The weekly rotation is transmitted to a threaded shaft projecting strictly within the axis of the cylindrical casing which makes possible perfectly symmetrical installation of the small motor in relation to the large drum which it drives. It should be noted that the cylindrical casing contains, in addition to the gear train, a friction safety device so that the drum can be rotated manually when attaching the paper without disconnecting the motor. The device also prevents damage to the gear train if the drum becomes blocked during operation for some reason.

Motor and drum are connected simply by placing them end to end so that the threaded shaft of the motor can be screwed directly into a threaded barrel drilled into the heavy metal shaft of the drum.

The assembly is sufficiently light so that it needs to be supported only by its central shaft. In a manner of speaking, it is suspended in air at the end of the axis of the drum where a simple footing consisting of a flexible spring prevents any rotation (which would otherwise be possible) of the stator of the motor in relation to the housing of the instrument and provides a certain elasticity in the connection.

An important point is the following: In order to prevent any possible interference by moisture, an electric heater with a consumption of 39 W has been placed in the housing between the drum and toward the front of the recorder, i.e., below the window with the recording slots for the two following purposes: (1) to reduce the moisture content of the air within the instrument so as to prevent any buckling of the paper due to moisture; (2) to prevent fogging of the recorder window by a slight hot-air current which has been proved effective in practice.

Another important remark is the following: Although this makes it possible to prevent fogging of the recorder window, this cannot be done in the same manner for the recess window. It is not possible to introduce in the immediate vicinity of the pendulum any source of heat, no matter how small, without the risk of causing considerable disturbances in the recordings. However, there is a solution for the case where such protection becomes absolutely necessary due to a particularly high moisture content of the ambient air. This is the solution of the "fog shield" ["parabuée"] in general use by astronomers to protect the objectives of astronomic telescopes during humid nights which is a simple tube prolonging the tube of the telescope beyond the objective.

For the horizontal pendulums, we adopted a similar solution by attaching at right angles to the window protecting the pendulum a sufficiently long tube, e.g., 25 cm, which may also be made of bakelite.

After several experiments, we found fog shields of bakelite covered on the outside by polystyrene foam, 1 cm thick, perfectly satisfactory.

An essential element of the recorder is the slot through which enters the light beam for recording on the photographic film. The width of this slot determines the fineness of the recorded trace so that the slot must be easily accessible for accurately adjusting the opening. The slot is therefore supported by a removable window consisting of a metal frame in which is mounted a glass window, 7 x 30 cm. Two beveled steel plates are affixed directly to the rear of the window by means of screws passing through the glass. The bevels face each other and leave between them the desired width of the slot. The bevels have been ground within a tolerance of less than 10 micron. By introducing small wedges of given thickness between the ends of the bevels, it is possible to provide a slot with a length of 30 cm and a width of 0.01 cm so that the latter will be constant within a few percent. In order to be able to adjust the width of the slot, the holes of the supporting screws in the glass are slightly oversize so that the two steel straight edges have a slight freedom of movement in relation to each other before the screws are tightened.

On the inner face of the glass window, lines spaced every 2 cm have been engraved perpendicular to the slot. Projected on the photographic film, these lines are printed on the paper and provide an excellent scale of reference which makes it possible, among others, to check any possible distortion of the film which might occur during developing or fixing. Moreover the scale is very useful when we desire to attach the end of one recording to the start of the following recording after the recording paper has been replaced.

The recorder is supported by three set screws for accurate adjustment of the horizontal position of the slot as well as the axis of rotation of the drum which is parallel with it.

The check of the drum opposite to the motor is provided with a large graduated circle. The movement of the graduations can be observed through a side window equipped with a locator mark. It is therefore always possible to check the position of the drum through observation from the exterior of the recorder.

The power-supply transformer of the low-voltage circuits, i.e., the circuit of the luminous hourly time marks and the circuit for the vertical and rectilinear filament of the light sources for the luminous spots also is within the recorder housing in order to be protected against moisture.

This position of the transformer makes it possible to standardize the installation of these electrical circuits by arranging a number of moisture-proof electrical connections on the front of the recorders which are connected by cable to the contacts of the clock, to the hourly time-mark lamps, to the luminous spots and to the 200-V network.

All circuits are protected by suitable fuses mounted on the recorder and a red pilot light on the front of the latter makes it possible to see in the dark whether the installation is energized or not.

It may be felt that these details have rather secondary importance but we should not forget that, in view of the awkward situation existing most of the time in the mine galleries, even the most elementary details which help to facilitate the work of the operator are also helpful for preventing errors in operation and for increasing the operating reliability of any installation.

Luminous signals

We need to make some reference here to the "luminous signals of the clinometric station": (1) The light beams which are illuminated every hour for a few seconds for recording the time marks on the photographic film; (2) the special lamps with vertical and rectilinear filament which are the sources of the light beam traversing the lens/mirror system of the pendulum and defining as real images of the filament the luminous spots which sweep the slot of the recorder.

The hourly light signals are very simple: two small light bulbs (6 V, 0.1 A) are mounted vertically on two supports on either side of the slot of the recorder and some 50 cm ahead of the latter. Their filaments (wound coil) can therefore be adjusted in a horizontal position at the height of the slot. The duration of the electrical contact of the pendulum and the intensity of the current of the two lamps must always be adjusted accurately so that the time marks on the photographic film will be very thin. The transformer in the housing is designed for multiple outlets so that we have a choice within a certain range of voltages (3.5-6.0 V) for the appropriate light of the lamps.

When the filaments are horizontal, their luminous projection through the slot may give a photographic trace with a thickness on the same order of magnitude as the width of the slot.

A certain complication arises from the presence of two lamps. They must evidently be strictly at the same height so that the traces printed separately by each luminous source coincide strictly. However, in view of the lateral position of each individual lamp, it is better to have two lamps if we desire to produce an hourly trace of constant intensity over all its length. However, it is possible to use only one lamp since the variation of intensity of the time mark does not produce any inconvenience during measuring.

The special lamps constituting the light sources of the luminous spots illuminating the mirror of the pendulum are lamps generally utilized for the reproduction of sound in movies and whose rectilinear filament remains always strictly taut (Phillips, type 3871-C, 6 V, 1.45 A). In order to prevent blackening of the photographic film by excessive ambient light, each lamp is enclosed in a cylindrical metal housing which is provided with a narrow slot not more than 1 mm wide parallel to the filament. This slot is directed toward the mirror of the pendulum from which we can therefore perceive the filament which is the source of the spot. We must here add an important remark. The lamp in question must be kept at an undervoltage so that it will not burn out within only a few weeks. This is due to the fact that the lamp is completely enclosed, the radiated heat cannot freely escape and the temperature within the lamp housing rises to more than 100° C so that the filament operates indirectly at an over-voltage with its rise in temperature. In order to provide for long life of the lamps, they must be operated at a definite undervoltage (the intensity selected is an average of 4 V at Sclaigneaux and Warmifontaine) while providing a normal intensity for the photographic recording. The lamps will have a long life since they are not subject to any vibration or shock.

The position of the reflected spot is adjusted both in height and laterally by suitably adjusting the position of the light source. This is arranged so that the rectilinear spot, i.e., the image of the filament, is cut in two by the slot. In order to facilitate this adjustment, a white screen is placed across the recorder window so that it leaves free only a narrow band on either side of the slot.

Due to slow drift, the luminous spot may leave the region of the slot. We must then activate the drift screw of the pendulum to return the spot to the proper position. In order not to have to make such an adjustment at too many intervals and to protect ourselves against a loss of the image due to an accidentally accelerated drift or even failure of one lamp, we provide a special arrangement by multiplying the light source and aligning them horizontally at intervals equivalent to one-half width of the slot and by adjusting them so that two images are printed permanently on the photographic paper. In that case, if a spot leaves the slot due to drift, it is immediately replaced by the image of the spot of the following light source, and if a lamp burns out, there is always a second spot on the slot.

We can also arrange matters so as to give them a slightly different luminous intensity which enables us to select the most appropriate image for measuring. Nor should we lose sight of the fact that the voltage is not always very stable in underground installations of this kind (unless we provide a stabilizer).

However, we should not have too many light sources because each of them is a source of thermal radiation which might eventually disturb the stability of the ambient temperature around the pendulum. On the other hand, we cannot make an exaggerated use of this means to correct a systematic drift because the azimuth in which the recordings of inclinations are made would change by too large a quantity. (At 5 m, the complete sweep of the 30-cm slot of our recorder corresponds to a change of azimuth of the pendulum arm of 3°).

For the ORB installation, we restricted ourselves to four light sources per recorder and have never noticed any difficulties of thermal origin.

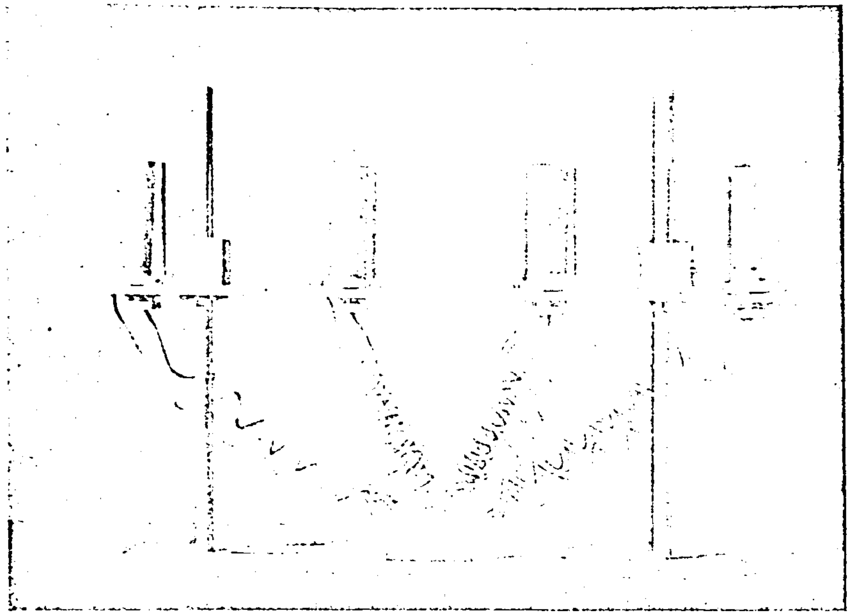


Fig. 45 - Adjustable support for spot projectors.

Special supports for the light sources aligned in groups of four have been constructed (Fig. 45) which guarantee stability of position of the lamps and to a certain extent facilitate adjusting operations because they allow the lamps to slide sideways and at the same time provide joint adjustment in height by sliding them along two vertical axes. The lamp housing can be adjusted so as to accurately project the light beam on the mirror.

Time-signal clock (Fig. 47)

In order to control the illumination of the hourly time-signal lamps, we need a clock with electrical contacts. We utilize a "Bürk" clock with weights electrically rewound and equipped with a wheel with removable contacts which can be placed every 5 min in accordance with the schedule selected. For the terrestrial tides, we place one contact at every full hour and an additional contact at 2330 hours (cf. Fig. 47) which helps in identifying the days and the numbering of the time marks after developing the film. This arrangement makes it easier to read the recording as will be seen in Figs. 39 and 40.

			1958											
			I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
STATIONS GRAVIMETRIQUES	UCCLE OBSERVATOIRE	G 145							2	3	3	3	3	3
	UCCLE OBSERVATOIRE	G 160												
	VEDRIN I.G.M.	G 98												3
	BATTICE I.G.M.	G 98												
STATIONS CLINOMETRIQUES	SCLAIGNEAUX OBSERVATOIRE	PH 1 (E-W)												
		PH 4 (N-S)												
		PH 9 (N-S)												
	WARMIFONTAINE OBSERVATOIRE	PH 11 (E-W)												
		PH 4 (N-S)												

			1959											
			I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
STATIONS GRAVIMETRIQUES	UCCLE OBSERVATOIRE	G 145	3	2			2	2	3	3	3	3	3	3
	UCCLE OBSERVATOIRE	G 160												
	VEDRIN I.G.M.	G 98		2	2									
	BATTICE I.G.M.	G 98												
STATIONS CLINOMETRIQUES	SCLAIGNEAUX OBSERVATOIRE	PH 1 (E-W)												
		PH 4 (N-S)												2
		PH 9 (N-S)												
	WARMIFONTAINE OBSERVATOIRE	PH 11 (E-W)												
		PH 4 (N-S)												

			1960												1961		
			I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	I	II	III
STATIONS GRAVIMETRIQUES	UCCLE OBSERVATOIRE	G 145	3		3	3	3	3	3	2	2	3	3	3			
	UCCLE OBSERVATOIRE	G 160				1	1	3	3	3	3	3	3	3			
	VEDRIN I.G.M.	G 98															
	BATTICE I.G.M.	G 98															
STATIONS CLINOMETRIQUES	SCLAIGNEAUX OBSERVATOIRE	PH 1 (E-W)				2	1		2	3	1	3	2				
		PH 4 (N-S)	1	2	1		3	2									
		PH 9 (N-S)								2	3	3	3				
	WARMIFONTAINE OBSERVATOIRE	PH 11 (E-W)															
		PH 4 (N-S)															

Fig. 46 - Recapitulation of activity of the Belgian earth-tide stations on 1 February 1961. The figures shown correspond to the number of harmonic analyses executed at each epoch by the Lecolazet method. An equal number of analyses by the Doodson-Lennon method was carried out except for the two gravimeters 145 and 160 in 1960. G: Askania gravimeters; PH: ORB horizontal pendulums. Uccle, Sclaigneaux and Warmifontaine are permanent stations of the Royal Belgian Observatory; Vedrin and Battice are temporary stations of the Military Institute of Geography.

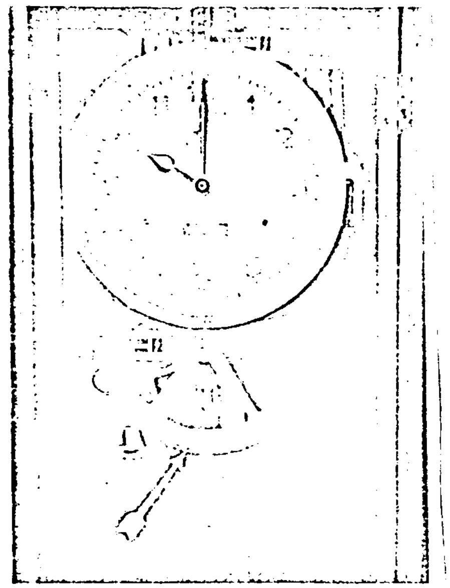


Fig. 47 - Bürk signal clock electrically wound. The arrow indicates the additional contact of 2330 hours on the contact wheel; we can also see most of the other contacts placed every hour.

It is better to place the clock outside of the observation chamber (or at a sufficient distance if it is installed in the gallery itself as at Sclaigneaux) because we then can make any necessary adjustments without disturbing the recording of the horizontal pendulums.

The clock must be adjusted so that it does not exceed a variation of 15 sec per week which is easy. In practice, it may obviously keep time less well.

An absolute rule for all earth-tide stations is to adopt "universal time" and this is the time to which the clock is set.

Constants and instrumental corrections

We have already seen above how calibration allows us to determine the fundamental constant K of the instrument.

We shall now discuss the details of the interpretation of the recordings obtained and will need to determine a number of constants which characterize the installation of each "horizontal pendulum/recorder" system. The instrumental constants and certain of their corrections are divided into two groups: the constants which provide interpretation of the amplitudes recorded and the constants which allow us to determine the dephasing (advance or retardation) of the oscillations in relation to the theoretical movement controlled by the moon and the sun.

a) Amplitude constants

After correct calibration, we know the constant K and consequently the sensitivity which corresponds to a period of 50 sec and a focal distance of 500 cm (distance of the pendulum lens to the recorder drum)(*).

Once the pendulum is in position, we must measure as accurately as possible the distance D to the recorder and the period of oscillation of the pendulum. The formula allowing us to transpose the measurements in millimeters on the recording paper A (mm) into milliseconds of arc A(mseca) is as follows:

$$A \text{ (mseca)} = \frac{D_0}{T^2 D} K.A \text{ (mm)} = \frac{500}{T^2 D} K.A \text{ (mm)}$$

Past experience has shown that the scatter of the measurements of amplitude of the principal semidiurnal lunar wave (M2) for our pendulums is on the order of 6-7 %. We must consequently consider that the determination of the factor $T^2 \times D$ must be made with greater precision, e.g., 1 %.

It is obvious that we can easily obtain a precision of 1 mm over the distance D which corresponds to 0.02 %. The period can be easily measured by means of a good chronometer and, by observing a series of consecutive oscillations (4 or 5) which the chonometer totalizes, we reduce the maximum error by as much. We can thus obtain a measurement within 0.1 sec which corresponds to a precision on the order of 0.14 %. Consequently, the relative error in the factor $T^2 \times D$ which is expressed by the combination $2 \frac{dT}{T} + \frac{dD}{D}$, will not attain 0.2 %. We saw earlier that the relative error on the constant K is on the order of 0.5 %. We can be well satisfied with this situation.

(*) We should point out that sensitivity is the relative variation of inclination from the vertical which corresponds to a displacement of 1 mm of the luminous spot on the sensitive paper.

b) Phase constants

1 - Azimuth of pendulum arm.

If the arm is very perceptibly directed in the N-S or E-W direction, a small error of azimuth practically does not affect amplitude (0.16 % for an azimuth of 5°; 0.64 % for 10°) but is integrally carried over into the phase.

This azimuth can be determined by autocollimation on the mirror of the pendulum arm but this requires us to know in its turn the angle which the arm makes with the plane of the mirror. The latter is not always very easy to determine. Moreover, in certain cases, there is an advantage in rotating the plane of the mirror by a small angle in relation to the arm in order to facilitate installation in the plane of the meridian or of the first vertical even if local conditions are not ideally suitable for this. For example, the planes of the mirrors at Warmifontaine make angles of $4^{\circ}30'$ and $94^{\circ}30'$ respectively with the pendulum arms.

2 - Parallax of time marks.

The method of printing time marks by brief illumination of the slot of the recorder as described above requires the rectilinear filament of the light source to be parallel to and at the same level as the slot.

It is difficult to accomplish this perfectly so that we must carefully determine the small correction necessary in the phases determined by harmonic analysis.

In order to do so, we must, with a chronometer in hand, interrupt one of the spots projected five minutes ahead of the full hour and re-establish it five minutes later. We can then control whether the time mark is perfectly centered in relation to the interruption of the curve. We must here take into account the thickness of the time mark (0.1-0.3 mm) in the calculation of this correction.

For example, the installation of the N-S component at Sclaigneaux required a correction of parallax of the mark of +0.105 mm. This corresponds to + 0.02 hour because the film speed is 5.3 mm/hr. The correction of the phases is found by multiplying this hour fraction by the hourly speed of each wave: on the average, the semidiurnal waves require a correction of $0^{\circ}7$ (speed on the order of 30°/hr) and the diurnal waves require an average correction of $1^{\circ}5$ (speed on the order of 15°/hr).

For the E-W component at Sclaigneaux, the correction was higher: 0.08 hour but the installation has not yet been modified in order not to introduce a break of continuity in the recordings. It is perfectly sufficient to apply the known correction to the phases (semidiurnal $\sim 2^{\circ}5$, diurnal $\sim 1^{\circ}2$).

3 - Clock correction.

At each weekly visit, the necessary adjustment of the clock is determined by utilizing a good chronometer timed against the hourly radio signals.

The scatter of the phases found for the phase of wave M2 is on the order of 1° which corresponds to 2 minutes of time. We can therefore be satisfied with a precision of 10 sec in the adjustment of the clock.

The clock is adjusted as soon as this correction reaches 30 sec.

It should be noted here that the duration of the contact also limits the precision but not yet in any appreciable manner because it is on the order of 10 sec per hour for the "Bürk" clocks.

4 - Phase identifications.

The sign to be assigned to the phases depends on a number of conventions which must be established at the start of the work.

In geodetics, the north and east components of the deviation from the vertical are designated as ζ and η; they are positive if the plumb wire or the bob of the horizontal pendulum are deflected towards south or west respectively. Such theoretical calculations as those of the homologous waves of

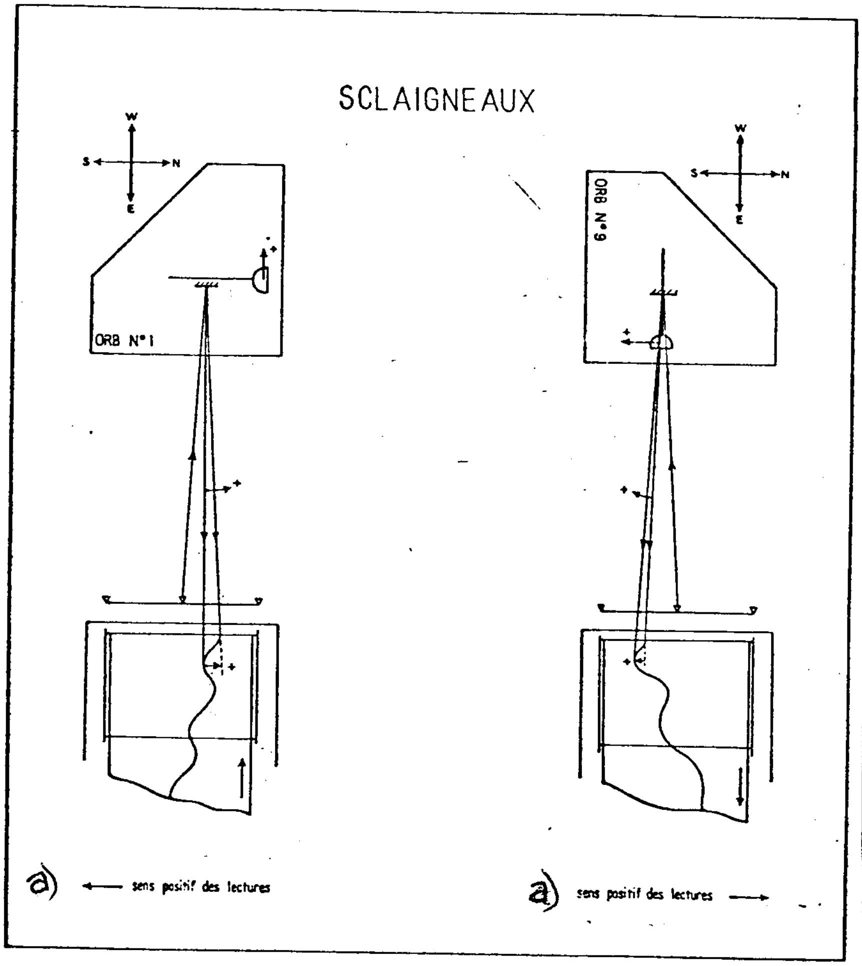


Fig. 48 - Orientation of the Sclaigneaux pendulums and their recordings in relation to the fundamental directions. The diagram is necessary for the interpretation of phase and drift.
Legend: a) positive direction of reading.

the Lecolazet method are based on this convention.

We must then check if the readings of the diagrams conform with the same convention. This is not necessarily the case by reason of the presence of a slight drift which determines the choice of a base line such that all the readings remain positive (which facilitates all operations of verification, perforation, and calculation).

We then establish a plan of the installation on the model of Fig. 48 where the positive directions of the theoretical calculation and the reading of the recordings are compared. For Sclaigneaux, we find that the existing drifts (cf. Figs. 39 and 40) have induced us to measure in the inverse direction in both cases. This results in an inversion of 180° of the phases found by calculation.

c) Accuracy of reading of recordings

Reading of the recordings is made in the form of measuring the ordinate at each hour as indicated by a time mark. We read within 1/10 mm as estimated by sight.

The principle adopted for determining the intrinsic precision of the recordings obtained from a horizontal pendulum or a gravimeter consists in eliminating from the curve of each day all the tidal waves (whose periods are accurately known) and a drift of higher order. The daily residue should be nil if the movement of the pendulum has been reliable and the measurement of the recording strictly accurate. However, the movement of the pendulum is disturbed by a number of secondary causes such as micro-earthquakes, thermal agitation, possible instrumental anomalies not well known whereas the measuring of the recordings cannot be made with all desirable precision because of the thickness of the mark and of certain slow microinclinations (periods on the order of 10-20 min) which we shall discuss further below. Consequently, there is a residue which we can calculate for each day of observation by applying a linear combination of ordinates which absolutely eliminates the drift and all tidal waves(*).

Such a combination leaving only systematic or accidental disturbances is called combination of errors.

At the International Center of Earth Tides, we have adopted a combination of errors suggested by R. Lecolazet (Bull. Inf. Marées Terrestres No. 17) and

(*) For the theory of the combination of ordinates, cf. "Analysis of the Graphics Resulting from the Superposition of Sinusoids" by H. and Y. Labrouste, Presses Univ. de France, 1943. A summary of the method was published in "Ciel et Terre", 1950, p. 78, under the signature of L. Couffigal.

have programmed its calculation on the IBM-650 electronic computer (Bull. Inf. Marées Terrestres No. 18).

With the notation of Labrouste, this combination is written

$$[E] = Z^5_{1/2}\, Z^6 \left(Y_3 - \frac{Y_0}{2}\right)$$

It applies to the 24 observations per hour of each day with the following 24 coefficients which are symmetrical in relation to 1130 hours: -1; -5; -10; 11; -10; 11; 10; -11; 10; -5; 1; 1; -5 ... -1.

This combination eliminates a polynomial of degree 5; its amplitude factor is strictly zero for waves with a period of 4, 6, and 12 hr and equal to 0.006 for waves with a period of 24 hrs.

With these conditions, the result of the combination depends only on such errors of observation as accidental or systematic disturbances of the pendulum, and possible errors of ~~reading~~ in measuring the recordings.

If this linear combination is carried out n times and each time for different data, we obtain n independent numbers ε_i from which we can derive the mean square error of an observation:

$$\sigma = \sqrt{\frac{\Sigma\, \varepsilon^2_i}{1872\, n}}$$

(in which 1872 is the sum of the squares of the coefficients of the combination).

We should draw attention to the advantage which consists in applying this combination of errors previous to any calculation of harmonic analysis. The computer successively punches the 30 values ε_i (on 5 cards) and then the sum of their squares and the value of σ. Examination of the 30 residues ε_i makes it possible to immediately detect the presence of an error of reading in the diagrams (which is sometimes difficult to find) and to localize it. The combination is especially effective in the case where a rather long part (several hours) of the recording has been difficult to read (micro-earthquakes) or has had to be interpolated. However, we note that certain individual values are affected only by the coefficient 1 whereas others are affected by the coefficient 11. Some of the possible errors therefore have a greater chance of being detected than others and the safest procedure is to apply the combination $[E_1]$ a second time but by staggering it by four hours.

A number of applications over 56 months of observation derived from five different Askania gravimeters always gave very close monthly values:

$$\sigma \cong 0{,}5 \text{ millimètre} \cong 2\ \mu\text{gals}$$

(in which μ gal = microgal 10^{-9} of the intensity of gravity).

For the ORB horizontal pendulums installed at Sclaigneaux, Warmifontaine and Bari (Italy), the value is less:

Sclaigneaux :	n° 1 (EW)	σ = 0,49 mm	= 0''0007	(période 66 sec)
	n° 4 (NS)	σ = 0,40 mm	= 0''0006	(période 60 sec)
	n° 9 (NS)	σ = 0,21 mm	= 0''0005	(période 51 sec)
Warmifontaine :	n° 11 (EW)	σ = 0,27 mm	= 0''0003	(période 73 sec)
Bari (*) :	n° 3 (EW)	σ = 0,15 mm	= 0''0004	(période 55 sec)
	n° 2 (NS)	σ = 0,18 mm	= 0''0004	(période 55 sec)

It should be noted that pendulum 1, the prototype of the series, is smaller and appears less satisfactory than the others. In regard to pendulum 4, it had a very evident optical defect which was corrected by substituting pendulum 9 for it. On the average we have therefore

$$\sigma \cong 0{,}3 \text{ millimètre} \cong 0''0005$$

This advantage in relation to the gravimeters is due without doubt to the systematic disturbances in these gravimetric recordings and caused by the thermostats or electronic amplifying devices.

We deduce from this that the error in the amplitude of wave M2 will be on the order of 0.14 μ gal or 0.3 % for the Askania gravimeters, and 0"000035 or 0.4 % for the ORB pendulums.

(*) Professor C. Morelli furnished us these excellent results and authorized their publication here.

The precision of the Askania gravimeters and of the ORB horizontal pendulums is consequently comparable and we may conclude that they can be satisfactorily associated in the measurement of earth tides if we want to arrive at a homogeneous representation of the phenomenon.

Stability in time of the instruments

The stability of a horizontal pendulum must be examined from two points of view which correspond to the two directions of adjustment determined by the period screw and the drift screw of the instrument.

In the pertinent literature, many articles have been devoted to the study of drift and we shall analyze in the following this aspect of the recordings obtained at Sclaigneaux and, in recent weeks, at Warmifontaine.

By contrast, little attention has been given to the period stability. However, the latter is of the greatest importance because it directly determines the coefficient of amplification of the instrument and affects any quantitative interpretation of the curves recorded. This is one of the specific difficulties encountered with the gravimeters whose sensitivity varies systematically and to such an extent that it seems necessary to adopt a specific coefficient for each day of observation.

Stability of the period

The constancy of the period of the horizontal pendulum depends on several factors.

In the first place, a satisfactory definition of the two points determining its axis of rotation is essential. In other words, these points must be permanent and cannot vary in position with the azimuth of the suspended pendulum which would modify the period in very appreciable proportions as a function of the azimuth and as much more appreciable as the periods are longer. In that case, the angle of the axis of rotation with the vertical is so small that the slightest displacement of the pivot point changes the angle and consequently the period by a proportionately very appreciable quantity.

We experimented with this by thickening the threads for several millimeters near the point of attachment. The position of the pivot point then becomes uncertain and fluctuates with the azimuth. We noted period variations of 75-90 sec for variations in azimuth of the arm of some 3°.

This is manifested even more markedly with the metallic pendulums where the wires are gripped by jaws. The effect must be even greater with bands. Its immediate consequence is the impossibility frequently encountered of exceeding a period of 30 or 35 sec with a metallic pendulum.

While attaching the quartz threads, care must therefore be exercised to make the weld short by reducing the original short connecting thread through capillary absorption so that the latter is finally constituted only by a minuscule mass of an approximately hemispherical form.

In the second place, the constancy of the period depends on qualities of the installation, i.e., on the stable anchorage of the pendulum in the terrestrial crust. This already indicated that our system of recesses and grouted steel bars produces very satisfactory results as shown by the table of period measurements below:

SCLAIGNEAUX

a) *Pendule 1*

		T	*S*
1960	mai 31	66,87	0"001474
	sept. 16	65,42	1536
	nov. 29	66,16	1506
1961	janv. 1	65,75	1525
	fév. 16	66,47	1492
	mars 22	67,35	1454
	avril 19	67,56	1444

b) Distance à l'enregistreur : 535,8 cm

a) *Pendule 4*

		T	*S*
1960	janv. 5	58,59	0"001687
	avril 4	60,34	1591
	juillet 12	62,04	1505

b) Distance à l'enregistreur : 472,0 cm

a) *Pendule 9*

		T	*S*
1960	sept. 13	50,50	0"002397
1961	janv. 18	51,98	2262
	fév. 16	52,88	2186
	(*) mars 22	57,73	1834
	avril 19	57,81	1829

b) Distance à l'enregistreur : 472,0 cm

Legend: a) Pendulum; b) Distance to recorder.
(*) Pendulum 9 was again adjusted on 25 February.

The three pendulums are type I with three set-screws. It is to be expected that the pendulums of type II with two set-screws will be still more stable (modified Nos. 11 and 4 at Warmifontaine). We sometimes hesitate to disturb the pendulum in order to obtain a period measurement during a weekly visit to the station and this explains in particular the few measures for No. 9. However, we do believe that the period should be measured every two months. It will be seen in particular that the period variations of No. 1 are small and that its sensitivity varied only for a maximum of 4 % in seven months. For comparison, the sensitivity of the Askania gravimeters 145 and 160 varied during the same interval of time by 12 and 7 % respectively.

On the other hand the metallic pendulums whose characteristics are given in literature show period fluctuations on the order of 3 % per month whereas their period does not exceed 30 sec.

The pendulums installed during recent weeks at Warmifontaine both have a period of 70 sec. We might work with even longer periods and without doubt on the order of 100 sec.

Although it is possible that the drift of azimuth is a function of the sensitivity, we have not yet experimented with this in order to provide for continuity of the observations. In order to determine this, two additional recesses have been cut in each of the stations and as soon as instruments are available, they will be installed here and adjusted to very high sensitivity.

Stability of the azimuth

The phenomenon of instability of the azimuth is commonly designated as drift.

Much research has been devoted to this phenomenon such as attempts at geophysical interpretation and discussion of the disturbing influence of drift in the determination of tidal waves.

The analytical methods utilized eliminate a parabolic drift for each interval on the order of one day but, as a precaution and for reasons of homogeneity, we always determined and eliminated the drift before carrying out an analysis itself.

The process employed is the application of a combination of ordinates developed by A. T. Doodson for the analysis of long-period ocean tides which can be slightly simplified, for earth tides, by reason of the fact that there are no subharmonics in this phenomenon.

Table 4 and Figs. 49, 50, and 51 make it possible to judge the importance of drift at the Sclaigneaux station.

A polynomial representation of drift was calculated with the aid of the IBM-650 computer. We thus found, with t expressed in days, 168 points between 1 April and 22 September 1960 for P_1(E-W)

$$E = E_0 + 0''017701\,t - 0''00012373\,t^2 + 0''000001615\,t^3 - 0''0000000048\,t^4$$

and 230 points between 25 November 1959 and 11 July 1960 for P_4(N-S)

$$E = E_0 + 0''023949\,t - 0''00009432\,t^2 + 0''000000747\,t^3 - 0''0000000019\,t^4$$

and 170 points between 24 August 1960 and 9 February 1961 for P_9(N-S)

SCLAIGNEAUX

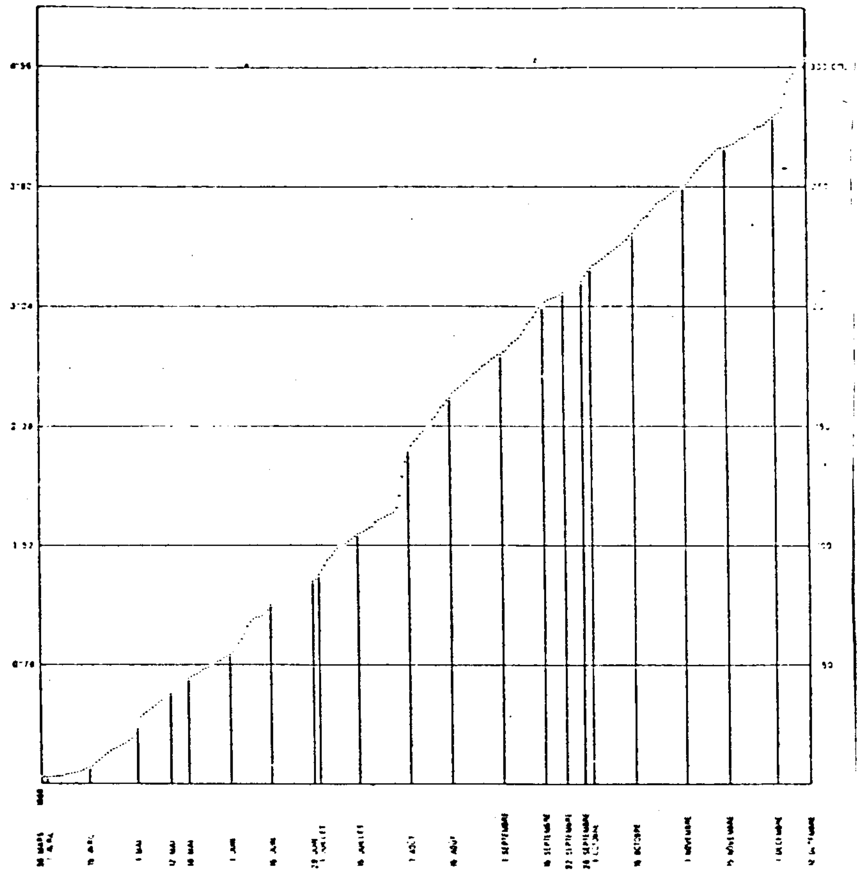


Fig. 49 - Drift toward east of ORB pendulum 1 at Sclaigneaux along E-W component during 256 days of observation.

The graphic representation of drift shows moreover that it is very nearly linear.

Pendulum 1 (E-W) drifts systematically toward east whereas pendulums 4 and 9 (which was substituted for No. 4 in July 1960) both drifted toward north.

It will be noted, however, that the importance of drift decreased considerably after a few months of operation. Since certain authors have stated the existence of variations of a seasonal character, it is difficult to already make a definite statement in regard to this point.

TABLE 4

Monthly drift noted at Sclaigneaux

Dates	E.W. component Pendulum 1		N.S. component Pendulum 4		Pendulum 9	
	cm	mseca	cm	mseca	cm	mseca
Dec. 1959			+ 35	+ 560		
Jan. 1960			+ 44	+ 704		
Feb.			+ 27	+ 432		
Mar.			+ 30	+ 480		
April	+ 20	+ 303	+ 38	+ 608		
May	+ 30	+ 455	+ 33	+ 528		
June	+ 34	+ 516	+ 15	+ 240		
July	+ 51	+ 774				
Aug.	+ 41	+ 622			+ 15	+ 357
Sept.					+ 12	+ 286
Oct.	+ 33	+ 501			+ 10	+ 238
Nov.	+ 29	+ 440			+ 10	+ 238
Dec. 1960					0	0

(1 mseca = 0"001).

However, it would seem that the drift noted at Sclaigneaux is in general less then that reported in stations equipped with standard metallic instruments.

Actually, the above formulas show that the daily drift is on the order of 0" 02 whereas the following figures can be found in literature: 0"04341 for one and 0"015 for the other pendulum at Winsford (England); 0"03 at Kounrad (USSR); less than 0"2 at Kondara (USSR); less than 0"2 for one and 0"0.05 for the other pendulum at Ashkabad (USSR). In a graphic published by the Bidston station during the symposium at Munich, we find a drift attaining 15" or 0"25 per day in 2 months for the N-S component and a daily drift on the order of 0"04 for the E-W component.

The probable causes of drift can be listed as follows:

1. Instrumental causes: Imperfect suspension; asymmetrical deformation of the support; deformation of the supporting screws in their nuts; slippage between screw and crapaudine; changes in cementing material of the steel bar

SCLAIGNEAUX

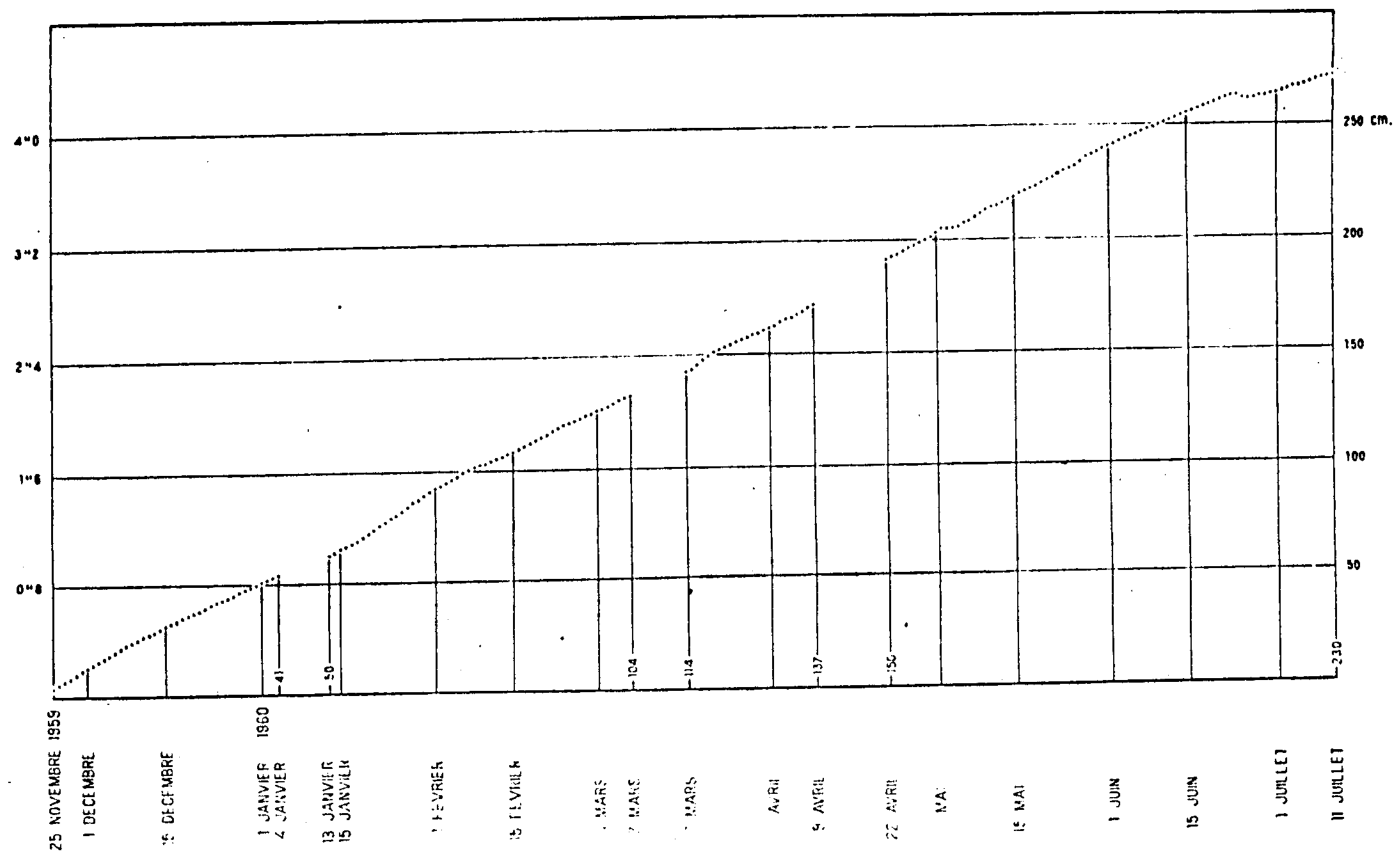


Fig. 50 - Drift toward north of ORB pendulum 4 at Sclaigneaux along N-S component during 230 days of observation.

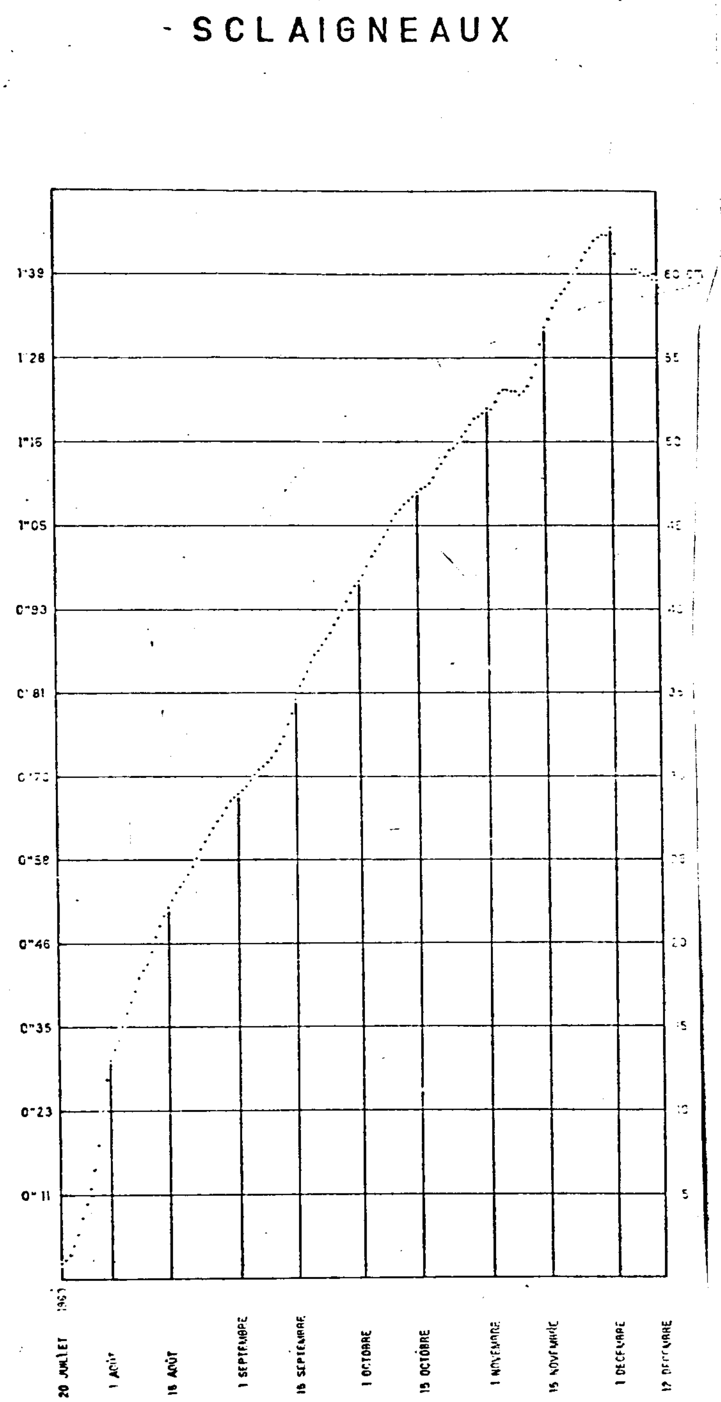


Fig. 51 - Drift toward north of ORB pendulum 9 at Sclaigneaux along N-S component during 144 days of observation.

in the rock; deformation of the supporting rock.

2. Geophysical causes: Periodic deformations due to variations of level in the underlying water table; instantaneous deformations of the crust under the effect of large zones of high or low pressure; deformation of the crust under the weight of snow or surface precipitation.

3. <u>Tectonic causes</u>: Bending stresses of the crust of the earth due to tectonic long-period phenomena affecting large sections of the crust of the earth.

In view of the multiple possible causes, we had to conclude that no definitive solution can be obtained through a single station equipped with a single pair of pendulums. Comparative examination of the residues of the polynomial representation of drift obtained from the computer for pendulums P_1 and P_4 and the Askania gravimeters 145 and 160 installed at the Observatory did not give any significant results so that we seem to be confronted by an aleatory phenomenon.

However, an accurate and unquestionable observation of a local geophysical effect has been made quite recently during the large and abrupt flooding of the Meuse River.

Within some twelve hours corresponding to the rapid rise of the water in the night from 31 January to 1 February 1961, <u>both</u> pendulums installed in the Sclaigneaux gallery at a distance of about 500 and 600 m from the river, drifted 10 to 15 cm where the combination of the two drifts manifested an inclination of the ground of 0"5 toward southeast, i.e., toward the river. This implies that under the weight of the water, the ground here assimilated to an enormous rigid slab in order to simplify the argument, dropped approximately 1 mm in the axis of the river. This is obviously only an order of magnitude and is even only the lower limit of the deformation because the latter is evidently not linear since the ground in reality curves under the influence of the overload.

We intend to resume discussion of this fact when accurate calculations will have been made for this phenomenon which occurred too recently.

The investigation of such effects from an overload on the ground are highly important because they make themselves felt over very large distances as soon as the overload is considerable. This can be seen from the fact that the periodic overloads on the continental platform by the masses of water carried by the ocean tides can be felt in the center of Europe, i.e., they introduce disturbing effects of the same period as the earth tides in the recordings of horizontal pendulums. They are called indirect effects and we shall see in Chapter V how we can attempt to separate them from the direct effects of the earth tides themselves.

It seems fairly obvious that the investigation of a phenomenon such as the flooding of the Meuse River might corroborate or eliminate certain theoretical concepts formulated in regard to this question.

This again shows the real scientific interest presented by the establishment of a network of clinometric stations.

Accordingly plans of the Royal Observatory for earth-tide stations provide four pendulums each for the two stations at Sclaigneaux and Warmifontaine.

Doubling of the instruments should make it possible to eliminate instrumental causes, doubling of the stations should make it possible to identify geophysical causes and then to derive conclusions in regard to tectonics.

Except for some unforeseen circumstances, the two stations will be fully installed at the end of 1961.

A first observation made at Warmifontaine is that the drift in time of the pendulums was shown to be definitely much less than at the Sclaigneaux station. During the first month of operation, the luminous spot of pendulum 11 (E-W) drifted only 5 cm which corresponds to a deviation of 0"05 in view of the focal distance of 5 m and the sensitivity of the instrument. Pendulum 4 drifted 8 cm or about 0"08 in 54 days.

We may assume that various improvements are the basis of the greater stability of the instruments. To begin with, pendulums 4 and 11 are type II where the right-angle screw has been eliminated and replaced by a rigid support (Fig. 23). Under normal conditions of installation and when the crapaudines are placed with sufficient accuracy, it is obvious that this screw is superfluous. It definitely produces greater stability of the instrument both in regard to constancy of period and consequently also of sensitivity and greatly reduced variation of azimuth (almost complete absence of drift). We should also take into consideration a very favorable additional factor at Warmifontaine which has moreover already been reported for stations abroad (e.g., Berchtesgaden), i.e., the great depth of the Warmifontaine station at 148 m below ground level whereas the Sclaigneaux station is protected only by a layer of ground not exceeding 80 m.

In addition, the recesses cut into the slatey schist at Warmifontaine are deeper and more uniform than those which could be cut into the dolomite at Sclaigneaux. At Warmifontaine, it is easier to insert completely the pendulums in the rock where a single panel for thermal insulation is sufficient to close the recess whereas two panels are necessary at Sclaigneaux in view of the geometric arrangement of the gallery.

Chapter IV

HARMONIC ANALYSIS OF RECORDINGS

Expansion of tidal potential into its principal waves

In order to make it easier for the reader to understand the significance of the results, let us briefly recall how the curve of the tides is broken down into a sum of harmonic components. We shall limit ourselves obviously to the principal waves which are those which can be more or less successfully derived from a minimum recording of one month.

Let us calculate the components of the lunar-solar attraction acting on the pendulum.

Let us consider the disturbing body L with mass m in Fig. 52 where the mass of the earth is taken as unity.

Let O be the center of gravity of the earth and A be the point of the terrestrial surface where we desire to investigate the effect of the attraction of L.

Let us select as axis OZ, direction of the vertical to A oriented toward the center of the earth, and OX perpendicular to the former at O in the plane LOA because the attraction of L is exerted in this plane at O and A.

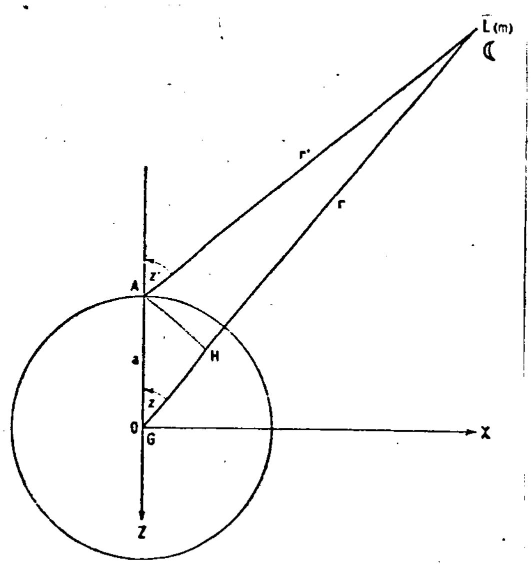


Fig. 52

Let us write the components of the attraction as O and A in accordance with these axes:

$$(1)\quad \begin{cases} Z_0 = -f\dfrac{m}{r^2}\cos z \\ X_0 = f\dfrac{m}{r^2}\sin z \end{cases}$$

$$(2)\quad \begin{cases} Z_A = g - f\dfrac{m}{r'^2}\cos z \\ X_A = f\dfrac{m}{r'^2}\sin z' \end{cases}$$

In which $g = f\dfrac{1}{a^2}$ is the acceleration of gravity at A (in the direction opposite to OZ). The components of the relative force acting on A therefore are

$$Z_A - Z_0 = \frac{f}{a^2}\left[1 - m\frac{a^2}{r^2}\left(\frac{r^2}{r'^2}\cos z' - \cos z\right)\right] \quad (3)$$

$$X_A - X_0 = \frac{f}{a^2}m\frac{a^2}{r^2}\left(\frac{r^2}{r'^2}\sin z' - \sin z\right) \quad (4)$$

The variation of intensity of gravity due to L is expressed by the variable section $Z_A - Z_0$.

The deviation from the vertical due to L is expressed by

$$\operatorname{tg} e = \frac{X_A - X_0}{Z_A - Z_0} \quad (5)$$

We shall now express these disturbances as a function only of the elements (r,z).

Fig. 52 shows that we can write:

$$r'\sin z' = r\sin z \quad (6)$$
$$r'\cos z' = r\cos z - a \quad (7)$$
$$r'^2 = r^2 + a^2 - 2ar\cos z \quad (8)$$

We know that $a/r = 1/60$ approximately in the case of the nearest heavenly body, i.e., the moon, so that we can neglect the small terms of higher order in the third power of a/r. We then obtain for the deviations from the vertical

$$e = \frac{3}{2} \frac{m}{\sin 1''} (\frac{a}{r})^3 \sin 2z \tag{9}$$

whereas the variable section of the intensity of gravity is

$$dg = - f m \frac{a}{r^3} (3 \cos^2 z - 1) = g m (\frac{a}{r})^3 (1 - 3 \cos^2 z) \tag{10}$$

These expressions of the horizontal and vertical component of the disturbance derive from the potential

$$W_2 = \frac{fm}{2} \frac{a^2}{r^3} (3 \cos^2 z - 1) \tag{11}$$

In order to verify this, it is sufficient to form the derivations

$$- \frac{1}{g} \frac{\partial W_2}{a \partial z} \qquad - \frac{\partial W_2}{\partial a} \tag{12}$$

We state

$$G = \frac{3}{4} f m \frac{a_1^2}{c^3}$$

in which m = relation of mass of moon to mass of earth; c = large semi-axis of the lunar orbit; a_1 = mean terrestrial radius.

We can also write

$$g_1 = f a_1^{-2}$$

and, from the numerical values presently established by astronomy and geodetics,

$$g_1 = 982{,}04 \text{ cm sec}^{-2} \qquad c = 60{,}27\ a_1$$
$$a_1 = 6.371{,}221 \text{ km} \qquad m = 1/81{,}53 \ (*)$$

we find

$$G = 26.206 \text{ cm}^2 \text{ sec}^{-2}$$

This fundamental constant for the theory of the tides is called the Doodson constant.

For the sun, we find

$$G' = 0{,}46051\ G$$

It is well known that the solar tides have one-half of the amplitude of those of the lunar tides.

(*) The ratio of the mass of the moon to that of the earth is the most unreliable of this total of numerical values. It would seem that we should now adopt m = 1/81.38 (cf. Pariiskii, Bull. Inf. Marées Terrestres, No. 23 - 1960)

The introduction of the concept of potential (lunar or solar) from which all the manifestations of the earth tides derive, allows us to limit the mathematical expansion to the study of this single function and we need only deduce from this the relative characteristics for each tidal effect through a suitable derivation.

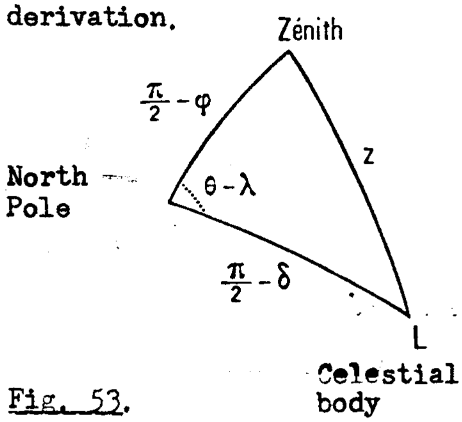


Fig. 53.

However, expression (11) of the potential W_2 cannot be solved because it introduces a local coordinate z of the disturbing heavenly body. We shall therefore substitute for it the customary equatorial coordinate (H = hourly angle; δ = declination) and the astronomic coordinates of the locus of observation (λ, ϕ) by using the triangle of position of spherical astronomy (Fig. 53)

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H \tag{13}$$

We then successively obtain

$$\begin{aligned}
\cos^2 z &= \sin^2 \phi \sin^2 \delta + \cos^2 \phi \cos^2 \delta \cos^2 H \\
&\quad + 2 \sin \phi \cos \phi \sin \delta \cos \delta \cos H \\
3 \cos^2 z - 1 &= 3 \sin^2 \phi \sin^2 \delta - 1 \\
&\quad + {}^3/_2 \cos^2 \phi \cos^2 \delta \, [\cos 2H + 1] \\
&\quad + {}^3/_2 \sin 2\phi \sin 2\delta \cos H \\
&= 3 \sin^2 \phi \sin^2 \delta - 1 + {}^3/_2 \cos^2 \phi \cos^2 \delta \\
&\quad + {}^3/_2 \cos^2 \phi \cos^2 \delta \cos 2H \\
&\quad + {}^3/_2 \sin 2\phi \sin 2\delta \cos H
\end{aligned}$$

The first of these terms may also be written

$$\begin{aligned}
&3 \sin^2 \phi \sin^2 \delta - 1 + {}^3/_2 (1 - \sin^2 \phi)(1 - \sin^2 \delta) \\
&\quad = {}^3/_2 [3 (\sin^2 \phi - {}^1/_3)(\sin^2 \delta - {}^1/_3)]
\end{aligned}$$

and finally

$$\boxed{\begin{aligned}
W_2 = G \{ &\cos^2 \phi \cos^2 \delta \cos 2H \\
&+ \sin 2\phi \sin 2\delta \cos H \\
&+ 3 (\sin^2 \phi - {}^1/_3)(\sin^2 \delta - {}^1/_3) \}
\end{aligned}} \tag{14}$$

The decomposition of the potential into three terms as in expression (14) is due to Laplace who was the first to show its remarkable significance and geometric characteristics. These three terms represent the 3 types of spherical harmonic surface functions of the second order described in the introduction (Fig. 1).

The expansion of the potential is the basis of the analysis and of the forecast of both earth and ocean tides.

The arguments of the components, very numerous even if we limit ourselves to the principal ones, of the tide are expressed as functions of the following six independent variables: τ = mean moon time; s = mean longitude of the moon; h = mean longitude of the sun; p = longitude of the lunar perigee; N' = - N in which N is the longitude of the ascending node of the moon; p_s = longitude of the perihelion.

In a first approximation, we assimilate the longitudes to the right ascensions (they are simultaneously equal to 0° and 90°).

The mean solar time is expressed as a function of the mean lunar time by the relation

$$\tau + s = t + h = \theta = \text{ sidereal time, i.e.,}$$

$$t = \tau + s - h$$

1) Sectorial function (semidiurnal waves).

The general expression of this first type of wave is

$$S = G.\cos^2\phi \cos^2\delta \cos 2H$$

where we state

$$G_s = G \cos^2\phi$$

Lunar waves

$$H = \tau$$

The lunar orbit is very complicated. We shall restrict ourselves here to describing its principal characteristics since only the most important waves are of interest to us.

The lunar orbit is inclined about 5° on the ecliptic and this inclination varies from $4^{\circ}59'$ to $5^{\circ}18'$ within 173 days. The declination of the moon consequently varies between the maximum limits + $28^{\circ}35'$ and $-28^{\circ}35'$ in 27.321 days (tropical month).

The mean value of the coefficient $\cos^2 \delta$ consequently is 1 and we can therefore write

$$\cos^2 \delta = 1 + n \cos 2s$$

Actually, this expression is more complicated; limited to its principal terms, it is written

$$\cos^2 \delta = 0{,}947 + 0{,}079 \cos 2s + 0{,}036 \cos (N - 180^{\circ}) + 0{,}36 \cos (2s - N)$$

Since the orbit of the moon is elliptic, the coefficient G_s must therefore be affected by a factor $(c/d)^3$ which will be of the form

$$1 + m \cos (s - p)$$

in which m = 0.164.

In principle, the form of S therefore is

$$G_s (1 + n \cos 2s) [1 + m \cos (s - p)] \cos 2\tau$$

or else

$$G_s \cos 2\tau + G_s m \cos (s - p) \cos 2\tau - G_s n \cos 2s \cos 2\tau - \\ - n m G_s \cos 2s \cos (s - p) \cos 2\tau$$

from which we separate all the individual components by systematically applying the elementary trigonometric relation

$$\cos A \cos B = \tfrac{1}{2} \cos (A + B) + \tfrac{1}{2} \cos (A - B)$$

We consequently have the following expansions:

$$\cos (s - p) \cos 2\tau = \tfrac{1}{2} \cos (2\tau - s + p) + \tfrac{1}{2} \cos (2\tau + s - p)$$
$$\cos 2s \cos 2\tau = \tfrac{1}{2} \cos (2\tau - 2s) + \tfrac{1}{2} \cos (2\tau + 2s)$$

$$\cos 2s \cos (s - p) \cos 2\tau = \tfrac{1}{4} \cos (2\tau - 3s - p) + \tfrac{1}{4} \cos (2\tau + s + p) \\ + \tfrac{1}{4} \cos (2\tau + 3s - p) + \tfrac{1}{4} \cos (2\tau - s + p)$$

The new waves are all distributed in pairs of arguments <u>equidistant</u> to that of the fundamental wave $G_s \cos^2 \tau$.

The significance of all these waves is the following:

1) $M2 \equiv G_s \cos^2 \tau$: dominant semidiurnal lunar wave called M2 by Darwin(*) which corresponds to the semidiurnal tide produced by an imaginary moon traversing a circular orbit in the plane of the equator at the mean velocity of the real moon.

Its period is therefore one-half of a lunar day or 12 hours and 35 minutes which corresponds to a speed per hour of $28^\circ984$.

2) $N2 \equiv 1/2\ G_s m \cos(2\tau - s + p)$ which are 2 "elliptic" semidiurnal lunar

3) $L2 \equiv 1/2\ G_s m \cos(2\tau + s - p)$ waves called N2 and L2 by Darwin(*).

Since $s - p = 0^\circ544$, the speeds per hour and the periods of these two waves are

for N2 : $v = 28^\circ 984 - 0^\circ 544 = 28^\circ 440$; $P = 12$ h 39 m
for L2 : $v = 28^\circ 984 + 0^\circ 544 = 29^\circ 528$; $P = 12$ h 11 m

These waves express the fact that the amplitude of the tides is greater when the moon is in the perigee and less when it is in the apogee. They correspond to the movement of imaginary bodies passing through the perigee at the same time as the imaginary moon of M2 ($s - p = 0^\circ$) and also when this moon is in the apogee ($s - p = 180^\circ$).

4) $K2m \equiv n/2\ G_s \cos 2(\tau + s)$: which is a declinational semidiurnal lunar wave.

$$2s = 1^\circ 098$$

so that the velocity per hour of K2 m is $28^\circ984 + 1^\circ098 = 30^\circ0821$ and its period is 11 hours 58 min which is one-half of the sidereal day because

$\tau + s = \theta$, i.e., sidereal time.

The calculation completely expanded from the complete expressions of $(c/d)^3$ and $\cos^2 \delta$ shows that the other waves have relatively low amplitudes which need not preoccupy us here.

(*) The mnemotechnical reason of the symbols adopted is the following: M2 recalls the moon and the periodicity of 2 tides per day; L2 and N2 are, from the point of speed, 2 waves symetrically adjacent to M2 so that the letters of the alphabet on either side of M were selected.

On the basis of the preceding remarks, it will be noted immediately that N2 and L2 theoretically should have exactly the same amplitude. In fact, this is not so because, due to the law of areas, the speed of the moon over its orbit varies at the same time as its distance from the center.

If we introduce this additional effect into our formulas, we obtain different amplitudes for N2 and L2. If the moon is near us, the amplitude of the tides should be greater. On the other hand, the speed of the moon over its orbit is, at that time, greater and the interval between two passages in the meridian is greater. Consequently this is the tide with the longest period, N2 which corresponds to the largest tides. N2 is the major elliptic wave and L2 is the minor elliptic wave.

Solar waves

$$H = t = \tau + s - h$$

The expansion is carried out in exactly the same manner but is on the whole simpler because the ellipticity of the terrestrial orbit is less and does not undergo disturbances as considerable as that of the moon.

1) $S2 \equiv G'_s \cos 2t = G'_s \cos (2\tau - 2h + 2s)$: which is the dominant semidiurnal solar wave called S2 by Darwin(*). It corresponds to the tide produced by an imaginary sun traversing a circular orbit in the plane of the equator at the mean velocity of the real sun.

Its periodicity consequently is one-half mean solar day or 12 hours and 00 minutes which corresponds to a speed per hour of 30° exactly.

2) R2, T2 are the major and minor elliptic solar waves(*). Their respective arguments consequently are

$$2t - (h - p_s) = 2\tau + 2s - 3h + p_s$$
$$2t + (h - p_s) = 2\tau + 2s - h - p_s$$

(*) The mnemotechnical reason of the symbols adopted here is the following: S2 recalls the sun and the periodicity of 2 tides per day; R2 and T2 are, from the point of view of speed, 2 waves symmetrically enclosing S2 so that the 2 letters of the alphabet on either side of S were selected.

3) K2s is the declinational solar wave and its period obviously also is that of a one-half sidereal day with its argument as

$$2(t + h) = 2\theta = 2(\tau + s)$$

It is therefore absolutely indistinguishable from the lunar wave K2m. We should consequently remember that this very special component is composed of a part of the lunar K2m and a solar part K2s which cannot be separated and where the lunar part represents 68 % of the amplitude.

We call "semidiurnal lunar-solar wave" the combination

$$(K\,2m + K\,2s) = K2$$

2. <u>Tesseral function (diurnal wave)</u>.

$$\begin{aligned} T &= G \sin 2\phi \sin 2\delta \cos H \\ &= G_T (c/d)^3 \sin 2\delta \cos H \end{aligned}$$

in which

$$G_T = G \sin 2\phi$$

<u>Lunar waves</u>

$$H = \tau; \quad \sin\delta \cong \sin\varepsilon \sin s; \quad \cos\delta \cong 1$$

In the case of the function T the factor sin 2 δ has as mean value 0 and we must therefore introduce expressions of the form:

$$\begin{aligned} \sin 2\delta &= 0 + 2 \sin\varepsilon \sin e \\ (c/d)^3 &\cong 1 + 0{,}164 \cos (s - p) \end{aligned}$$

It follows directly from this that the coefficient of the fundamental wave of argument τ (speed per hour $14^{\circ}492$) and which would be called M1 is zero. Consequently there is no fundamental diurnal wave analogous to M2 and corresponding to the same imaginary moon because this body circulates in the equator ($\delta = 0$, $\sin 2\delta = 0$).

Accordingly, there are no diurnal waves which are declinational and elliptic as such:

K1m = declinational wave(*) of argument $(\tau + s) = \theta$:

$$14^{\circ}\,492 + 0^{\circ}\,549 = 15^{\circ}\,041$$

(*) As before, the symbols adopted have a mnemotechnical reason: K1 and O1 are symmetrical to M1 in the tide and in the alphabet.

O1 = declinational wave(*) of argument $(\tau - s)$:

$$14°492 - 0°549 = 13°943$$

which would be of the same amplitude in order to exactly compensate each other and give 0 when the declination of the celestial body is zero. (However, this is still only an approximation and thus makes O1 slightly superior in amplitude to K1). The period of K1m is 1 sidereal day.

Two elliptic waves of O1:

Q1 of argument $(\tau - s) - (s - p) : 13°943 - 0°544 = 13°399$
ε(O1) of argument $(\tau - s) + (s - p) : 13°943 + 0°544 = 14°487$

Two elliptic waves of K1m:

J1 of argument $(\tau + s) + (s - p) : 15°041 + 0°544 = 15°585$
ε(K1m) of argument $(\tau + s) - (s - p) : 15°041 - 0°544 = 14°497$

We note immediately that the two waves provisionally designated by ε (O1) and ε (K1) have speeds per hour very close together because their arguments differ only by 2p. In fact, they differ extremely little from the vanished fundamental wave M1 and their combination is consequently designated by this symbol.

Solar waves

$$H = t = \tau + s - h$$

The imaginary sun circulating in the equator at zero declination, there consequently is no fundamental diurnal solar wave S1 whose period would be exactly one full mean solar day and whose speed per hour would be $15°$.

The diurnal solar waves will also be exclusively declinational and elliptic:

K1s = declinational wave of argument t + h = θ:

$$15°000 + 0°041 = 15°041$$

(*) As before, the symbols adopted have a mnemotechnical reason: K1 and O1 are symmetrical to M1 in the tide and in the alphabet.

Pl = declinational wave of argument t - h:

$$15^\circ 000 - 0^\circ 041 = 14^\circ 959$$

The wave Kls whose period is one full sidereal day consequently combines with Klm by forming in a diurnal <u>lunar-solar</u> wave in a manner analogous to K2.

Two elliptic waves of Pl:

$\pi 1$ of argument $(t - h) - (h - p_s)$: $14^\circ 959 - 0^\circ 041 = 14^\circ 918$

$\varepsilon(P1)$ of argument $(t - h) + (h - p_s)$: $14^\circ 959 - 0^\circ 041 = 15^\circ 000$

Two elliptic waves of Kls:

$\psi 1$ of argument $(t + h) + (h - p_s)$: $15^\circ 041 + 0^\circ 041 = 15^\circ 082$

$\varepsilon(K 1 s)$ of argument $(t + h) - (h - p_s)$: $15^\circ 041 - 0^\circ 041 = 15^\circ 000$

We find again that the waves ε(Pl) and ε(Kls) combine into one wave which has the same speed as the vanished fundamental wave Sl (they differ only by $\pm p_s$ whose period is 20,000 years!). It is therefore given this name.

The value of the Doodson fundamental constant G and the strict expansion of formula (14) which we were only able to sketch here, allow us to calculate the amplitude of each of the waves for any given station.

Of course, these amplitudes also depend on the azimuth in which the pendulum operates and this is already assumed from examination of Fig. 1. We see here specifically that the E-W component never includes any long-period terms (zonal function) and that, at the latitude of 45°, the N-S component does not include any diurnal waves (they are here maximum in the vertical component so that the drift in the N-S direction is zero).

Formula (9) is written, by projection on the cardinal points,

$$F_s = C \sin 2z \cos A$$
$$F_w = C \sin 2z \sin A$$

where the azimuths are considered as positive from south to west. However,

$$\cos z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos H$$
$$\sin z \cos A = - \cos \varphi \sin \delta + \sin \varphi \cos \delta \cos H$$
$$\sin z \sin A = \cos \delta \sin H$$

which immediately gives us

$$F_s = C\{{}^3/_2 \sin 2\varphi\,({}^1/_3 - \sin^2\delta) - \cos 2\varphi \sin 2\delta \cos H + {}^1/_2 \sin 2\varphi \cos^2\delta \cos 2H\}$$

$$F_w = C[\sin\varphi \sin 2\delta \sin H + \cos\varphi \cos^2\delta \sin 2H]$$

The geographic factors are grouped in the table below which is useful for calculating the theoretical amplitudes of the various waves:

Component	Long period	diurnal waves	Semidiurnal
Vertical	$\frac{1}{2}(1 - 3\sin^2\varphi)$	$\sin 2\varphi$	$\cos^2\varphi$
North-South	${}^3/_4 \sin 2\varphi$	$\cos 2\varphi$	${}^1/_2 \sin 2\varphi$
East-West	0	$\sin\varphi$	$\cos\varphi$

For Sclaigneaux, we derive from this the theoretical values of Table 5 and those of Warmifontaine differ very little from this.

Method of analysis for extracting the principal tidal components from the observed oscillations

The methods utilized for the investigation of the earth tides utilize successive stages in the filtering of the several waves. The first operation consists in eliminating the drift and by reason of this, the long-period waves (zonal tides) of the recorded curve.

The curve is read every hour. On this monthly table of 744 measurements (31 days) are carried out the calculations which lead to the determination of the amplitudes and phases of the principal waves, i.e., the diurnal waves K1, O1, Q1, M1, J1 and the semidiurnal waves M2, S2, N2, L2.

The first operation consists in replacing the table of 744 numbers (31 x 24) by a table with 33 lines and 2 columns (Lecolazet method) or a table with 29 lines and 4 columns (Doodson-Lennon method). In these new tables, the group of diurnal waves is separated from the group of semidiurnal waves. They are obtained by multiplying the initial table line by line by vectors appropriately selected through the theory of combination of ordinates. These vectors are therefore staggered from 24 to 24 hours in the Doodson-Lennon method but from 21 to 21 hours in the Lecolazet method.

Table 5

Theoretical amplitudes of the principal waves for the Sclaigneaux station derived by the Doodson method.

Principal waves	K1	O1	Q1	M2	S2	N2	L2	ƒ	P
Mean amplitudes				E-W Component					
in mseca	7.071	5.024	0,962	10.045	4.685	1.923	0.276		
in mm	4.714	3.349	0,641	6.697	3.123	1.282	0.184	1.5	1
				N-S Component					
in mseca	1.682	1.195	0,229	7.729	3.605	1.480	0.212		
in mm	1.051	0.746	0,143	4.830	2.253	0.925	0.132	1.6	4
	0.731	0.519	0,099	3.360	1.567	0.643	0.092	2.3	9

for the E-W component (P1); 1 mm = 1.5 mseca
for the N-S component (P4); 1 mm = 1.6 mseca
for the N-S component (P9); 1 mm = 2.3 mseca

If we make these calculations without mechanical aid, we utilize perforated stencils which carry the appropriate multiplier opposite the perforations. The stencils are then placed on the table and the calculation is carried out with the numbers showing in the windows. These stencils are staggered by 24 or 21 hours depending on the method utilized in calculation. An example of these stencils can be seen in Fig. 54.

The results of this operation are subsequently subjected to new combinations by vectorial multiplication. However, at the end of these operations, the two methods differ in concept.

In the Lecolazet method specifically intended for investigation of the earth tides, the author operates on the whole of the theoretical components of amplitude above the nanogal (10^{-9} gal = 10^{-12} G) a combination which restitutes the wave amplitudes and phases homologous with those which the process applied to the observations isolated, and which in fact is not sufficiently selective for correctly separating all these waves with extremely closely adjacent periods. In this calculation, Lecolazet takes into account 52 diurnal and 27 semidiurnal waves.

The comparison between observation and theory is thus made by a very strict procedure.

In the Doodson method particularly adapted to the determination of the constants of ocean tides, the author carries out on the results of calculation of the reduction of the observations, a calculation of second approximation for eliminating the contribution of the secondary waves (P_1, φ_1, ψ_1 ..., R_2, T_2, λ_2 ...) in the principal waves. The number of components here taken into account is much less than that of the Lecolazet method.

Calculation in practice

The calculations of harmonic analysis in practice of the earth tides is not easy and requires a great deal of time. However, the authors of the

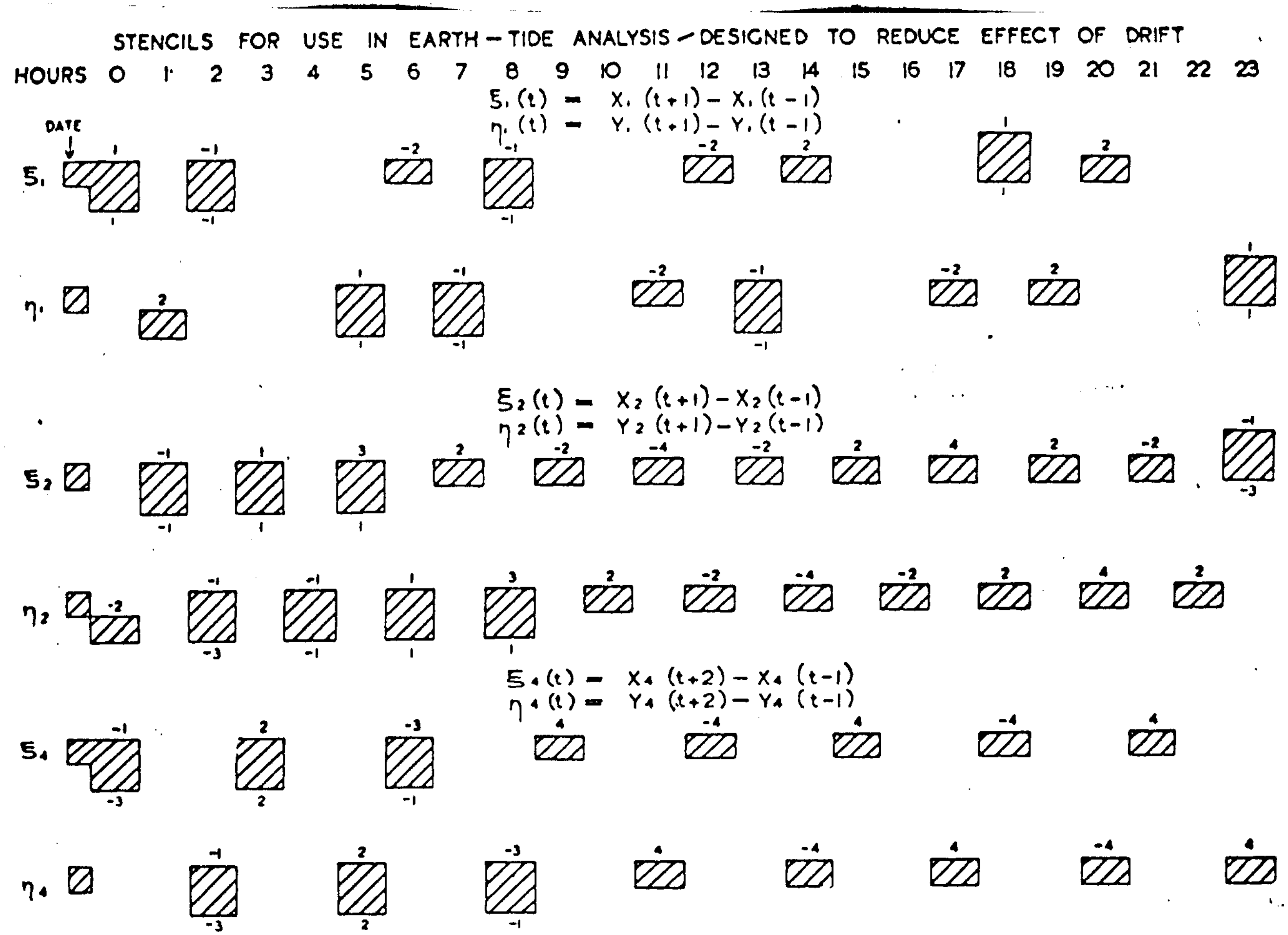


Fig. 54 - **Stencils representing combinations of ordinates in the Doodson-Lennon method:** ξ_1, η_1, **for diurnal waves;** ξ_2, η_2 **for semidiurnal waves,** ξ_4, η_4 **for the subharmonics of the fourth order.**

analytical methods have attempted to simplify the coefficients and to systematize the work in order to facilitate as much as possible the task of the operator and devised for this purpose the perforated stencils mentioned above.

It remains true nevertheless that these are only half-measures and that numerous controls are necessary if we want to obtain reliable results. In most cases, moreover, there is no other possibility than to repeat calculation. It is not difficult to understand the time lag which these conditions introduce in obtaining the results -- amplitude and phase -- suitable for geophysical interpretation.

The solution today consists of the large electronic computers and one of the authors programmed, for the IBM-650, the various stages of the harmonic analysis of the tides according to the methods of both Doodson and Lecolazet(*).

All readings of the diagrams are perforated on cards, 7 to each card or about 100 cards for one month of observation which the machine reads and transmits to the memory in 30 sec. Reading, calculation and elimination of the drift (which has been shown in Figs. 49, 50, 51) takes a total of 3 minutes per month and the detection of possible abnormal readings takes 30 seconds (at reading speed). Calculation by the Doodson and Lecolazet methods requires 3 minutes and 1 minute respectively for the stencils and the final calculation of amplitude and phase observed for the principal waves takes 2.5 and 2 minutes respectively. Finally, the calculation of the homologous waves of Lecolazet requires 7 minutes.

We thus see that a complete examination of a month of observations by two very different methods requires not more than 19 minutes.

A human mathematician equipped with a good electric calculator would have required approximately one month for making the same calculations.

(*) P. Melchior: Programmation des diverses méthodes d'analyse harmonique sur ordinateur électronique I.B.M. 650. (Bulletin d'Inf. Marées Terrestres, no. 15, pp. 249-255, no. 16, pp. 262-265, no. 17, pp. 288-290, no. 18, pp. 307-310, no. 19, p. 325).

For over a year, already a great number of results have been obtained from the recordings regularly made at Sclaigneaux. The values derived for the amplitudes and phases of the several waves of the tide are shown in Tables 6, 7 and 8. They were calculated from recordings carried out by means of 3 different ORB pendulums.

These tables do not show the absolute values of the amplitudes and phases found for each wave but values which must be considered relative and are obtained from comparison with the theoretical values derived by celestial mechanics as we shall see.

We thus find in these tables: the ratio γ = observed amplitude/theoretical amplitude; and the difference α = theoretical phase - observed phase, by assigning to each value of α a + or - sign depending on whether the observed phase is behind or ahead of the theoretical phase.

The results for the principal waves (amplitudes greater than 3 mseca) are in heavy print, i.e., K1, O1, M2, S2, for the E-W component and M2, S2 for the N-S component.

To each interval of 29 days have been applied the two methods of Doodson-Lennon (D) and of Lecolazet (L) but the mean interval concerning each of these two calculations may differ by one or two days. The results are grouped so that we can render account of the relatively small differences between the two calculation procedures.

The results obtained since the establishment of the station at Sclaigneaux present a remarkable stability as is shown by Tables 6, 7 and 8. We might almost say that previously no results with as little scatter have been obtained by means of horizontal pendulums.

Translator's note: [Tables 6 to 8]

"Composante" - Component; "Facteur" - Factor; "Epoque" - Time; "Pendule" - Pendulum; "Suite" - continued.

TABLE 6

Composante E.W.

Pendule O.R.B. n° 1

Epoque			FACTEURS γ						
			K1	O1	Q1	M2	S2	N2	L2
1960	IV	14	0,724	0,754	0,854	0,972	0,851	0,891	1,487
		14	0,646	0,704	0,692	0,967	0,721	0,962	1,348
	IV	26	0,732	0,766	0,728	0,957	0,812	0,914	2,027
		27	0,761	0,730	1,002	0,970	0,857	0,885	1,169
	VI	1	0,833	0,574	0,887	0,911	0,752	0,846	0,026
		2	0,824	0,679	1,192	0,911	0,719	0,955	0,521
	VII	13	0,909	0,629	0,594	1,008	0,708	0,923	2,681
		14	0,888	0,571	0,310	1,011	0,924	0,891	1,264
	VII	23	0,955	0,667	0,368	1,010	0,937	0,919	2,390
		24	0,946	0,621	0,635	1,026	0,940	1,034	1,265
	VIII	2	0,963	0,644	0,638	1,002	0,991	0,933	1,558
		4	0,946	0,580	0,807	0,997	0,992	1,109	0,779
	VIII	12	1,004	0,653	0,916	0,995	0,978	0,917	1,129
		14	0,997	0,624	1,029	1,016	0,959	0,988	0,526
	VIII	22	1,006	0,589	0,634	1,016	0,964	1,203	0,635
		25	0,974	0,568	1,029	1,016	0,948	1,027	0,433
	IX	1	1,024	0,658	0,956	1,022	1,029	1,202	0,977
	X	13	0,783	0,590	0,925	0,939	0,951	0,831	1,875
		13	0,750	0,594	0,678	0,950	0,945	0,960	1,643
	X	23	0,794	0,576	0,185	0,911	0,874	0,863	2.166
		23	0,769	0,629	0,402	0,919	0,863	0,952	3,174
	XI	2	0,750	0,628	0,681	0,884	0,921	0,922	1,706
		2	0,747	0,728	0,361	0,888	0,925	0,900	1,243
	XI	12	0,734	0,693	0,888	0,874	0,836	0,723	0,503
		12	0,698	0,770	0,581	0,876	0,821	0,671	0,274
	XI	22	0,656	0,634	0,773	0,893	0,913	0,648	0,913
		23	0,623	0,649	1,010	0,903	0,913	0.680	0,422

TABLE 6 (suite)

Composante E.W.

Pendule O.R.B. n° 1

PHASES							
K1	O1	Q1	M2	S2	N2	L2	*Méth.*
+ 7°95	— 6°33	— 3°79	—5°37	— 2°46	+ 1°81	— 4°94	D
+ 2°23	— 9°10	— 9°06	—6°85	— 3°01	— 1°71	+ 0°71	L
— 0°81	— 0°27	+20°19	—5°48	— 0°85	— 0°57	—24°53	D
— 3°61	+ 4°06	+34°03	—5°60	— 1°01	— 1°69	—29°03	L
— 5°53	+13°07	—59°93	+2°80	+15°97	—15°60	+10°13	D
— 9°32	+ 1°75	+49°86	+1°04	+ 9°09	— 0°63	+ 7°74	L
— 9°19	—13°00	—	—4°12	+ 8°26	— 8°85	—19°45	D
—13°42	—11°44	—34°27	—3°65	+10°98	— 8°55	—32°55	L
—11°29	— 0°63	—72°07	—4°00	+ 3°87	—10°37	—26°48	D
—14°65	— 0°83	—62°11	—5°04	+ 5°33	—13°32	—10°33	L
— 7°56	+ 4°34	—27°42	—4°60	+ 1°77	— 4°45	—11°94	D
— 9°04	— 6°16	— 1°82	—4°38	+ 3°93	— 5°20	—17°50	L
— 5°58	— 8°91	—11°28	—3°86	+ 2°25	—10°23	— 4°01	D
— 7°91	—16°16	— 2°15	—4°05	+ 3°78	— 1°32	— 5°77	L
— 3°35	— 8°34	—34°59	—4°17	+ 1°67	—10°37	+ 9°02	D
— 7°68	—10°80	— 7°79	—3°11	+ 4°60	+ 1°23	+ 8°90	L
— 2°88	— 4°73	—	—2°36	+ 1°48	—20°62	+55°24	D
+ 5°77	— 8°98	—22°50	—5°73	— 3°56	+ 1°32	—	D
+ 5°48	— 6°85	— 6°33	—6°03	— 2°04	— 0°42	+11°94	L
— 1°91	—11°74	+66°55	—5°67	— 0°96	+ 5°68	—	D
+ 0°76	—13°23	—27°97	—6°85	+ 0°84	+ 6°96	41°52	L
— 0°89	— 8°54	—	—4°59	— 1°27	— 5°66	—	D
+ 1°68	—11°81	—23°27	—4°78	— 0°40	— 8°99	+48°03	L
+ 2°25	— 5°00	+65°72	—3°56	+ 1°30	—10°13	—	D
+ 1°56	—18°63	+32°74	—3°51	+ 2°22	— 6°00	+38°86	L
+ 1°68	— 9°57	+25°90	—4°01	— 1°31	—13°83	+40°83	D
— 1°41	+ 8°68	+ 5°31	—4°64	— 1°40	— 6°34	—60°55	L

TABLE 7

Composante N.S.

Pendule O.R.B. n° 4

Epoque			FACTEURS γ						
			K1	O1	Q1	M2	S2	N2	L2
1959	XII	11	2,219	2,849	4,821	0,899	1,078	0,942	1,757
		10	2,202	2,458	4,878	0,894	1,076	0,984	0,815
	XII	19	2,115	2,640	3,688	0,905	1,095	0,970	1,019
		19	2,063	2,289	3,925	0,898	1,083	1,010	0,947
1960	I	28	2,207	2,731	2,899	0,890	0,980	0,870	0,813
		27	2,298	2,527	3,072	0,921	1,000	0,927	1,655
	II	8	2,330	2,615	2,847	0,892	0,995	0,874	1,167
		9	2,363	2,565	1,033	0,898	0,971	1,025	1,075
	II	19	2,463	2,349	3,615	0,888	0,988	0,971	1,162
		22	2,689	2,085	5,006	0,883	0,924	1,016	1,445
	III	27	1,887	2,742	3,117	0,908	0,975	0,669	1,188
		28	2,153	2,525	3,645	0,939	0,932	0,699	3,873
	V	17	2,483	2,732	1,670	0,954	0,999	0,909	1,614
		17	2,493	2,604	2,551	0,936	1,047	1,048	0,483
	V	27	2,429	2,600	4,785	0,944	1,001	1,022	1,773
		28	2,312	2,749	5,396	1,012	0,945	1,045	6,672
	VI	6	2,389	2,970	4,777	0,933	1,020	1,000	1,123
		7	2,382	2,823	4,705	0,980	0,975	0,976	0,830
	VI	16	2,391	2,552	6,527	0,938	1,021	0,947	1,560
		18	2,417	2,523	6,828	0,990	0,959	1,006	1,192
	VI	26	2,072	1,968	5,091	0,948	1,076	0,892	1,266
		27	2,506	2,366	6,023	0,937	1,044	1,092	0,734

TABLE 7 (suite)

Composante N.S.

Pendule O.R.B. n° 4

PHASES							
K1	O1	Q1	M2	S2	N2	L2	*Méth.*
—15°56	—17°07	—18°77	—3°51	— 5°01	+ 3°66	—48°19	D
—18°22	—15°91	—18°75	—2°96	— 4°66	+ 2°67	—38°07	L
—18°50	—21°69	—29°95	—4°05	— 7°40	+ 0°80	—41°95	D
—18°22	—22°28	—23°66	—3°84	— 7°12	— 0°27	—15°77	L
— 4°26	— 8°40	+76°18	+2°66	— 0°54	+ 6°52	—18°46	D
—10°93	—13°86	+34°29	+3°35	— 0°64	— 1°08	— 6°78	L
—12°50	—12°14	— 7°00	+1°86	— 0°23	+ 6°67	+ 8°13	D
—15°26	—17°44	+53°40	+2°58	+ 0°72	+ 7°03	—12°38	L
—15°19	—11°71	—19°63	+2°34	— 1°72	+ 8°04	+21°93	D
—12°46	—15°56	+ 0°24	+3°06	— 0°81	+ 5°35	+ 9°98	L
—11°00	—13°31	— 9°53	—0°31	— 1°26	+11°25	—	D
— 2°54	—11°28	+21°86	—0°19	—	+ 2°55	+42°38	L
—12°80	— 5°22	—23°05	+2°84	+ 1°55	+10°47	—18°93	D
—11°39	— 7°76	— 8°31	+3°44	+ 2°09	+ 4°25	—12°32	L
—14°00	—15°00	—12°71	+1°02	— 0°71	+32°13	—32°11	D
—15°32	— 1°18	+ 9°60	+2°03	+ 1°43	+ 5°77	—29°16	L
—12°00	—18°80	—41°55	+0°01	— 0°05	+ 8°91	—	D
—13°50	—17°37	—34°26	+1°25	— 1°32	+11°63	+33°06	L
— 9°92	—18°82	—25°30	+1°07	+ 1°48	+ 7°63	+12°58	D
—12°34	—18°89	+ 6°90	+1°56	— 1°84	+15°00	+35°63	L
—10°42	+ 3°46	+36°50	+0°03	+ 0°17	+ 5°65	+13°66	D
— 9°38	—15°80	—13°01	+0°79	+ 2°40	+12°71	— 0°72	L

TABLE 8

Composante N.S.

Pendule O.R.B. n° 9

Epoque			FACTEURS γ						
			K1	O1	Q1	M2	S2	N2	L2
1960	VIII	4	2,142	2,851	2,456	0,956	1,099	0,931	1,461
		4	1,987	3,069	1,718	0,940	1,132	0,903	1,463
	VIII	14	1,572	3,220	8,082	0,940	1,048	1,117	1,040
		14	1,988	2,869	2,571	0,924	1,059	0,999	0,576
	VIII	24	2 064	2,477	2,242	0,946	1,077	0,848	1,234
		25	2,106	2,888	1,576	0,917	1,071	0,997	1,224
	IX	3	2,026	2,537	4,336	0,921	1,065	1,049	1,582
		4	2,075	2,697	3,361	0,921	1,083	1,074	1,026
	IX	13	2,073	2,733	3,817	0,926	1,049	0,945	0,775
		15	2,014	2,727	3,375	0,933	1,055	1,014	0,492
	IX	23	1,905	2,676	3,573	0,955	1,089	0,982	0,250
		25	1,798	2.731	2,786	0,952	1,099	0,957	1,407
	X	3	1,924	2,654	3,280	0,966	1,060	0,946	0,809
		6	1,749	2,695	2,907	0,962	1,075	1,016	1,051
	X	13	1,940	2,743	2,889	0,960	1,082	0,978	1,283
		16	1,666	2,715	2,743	0,957	1,103	1,105	1,682
	X	23	1,871	3,054	3,334	0,940	1,085	0,987	0,944
		27	1,837	2,708	4,599	0,922	1,080	1,145	1,338
	XI	2	1,668	2,579	6,359	0,921	1,064	0,991	0,896
		6	1,989	2,698	4,379	0,910	1,073	1,109	0,862
	XI	12	1.885	2,813	5,949	0,922	1,047	1,020	1,017
		17	2,216	2,682	4,002	0,912	1,038	1,137	0,650
	XI	22	2,124	3,022	1,494	0,933	1,096	0,949	1,281
		27	2,190	2,931	2,470	0,912	1,025	1,122	0,361

TABLE 8 (suite)

Composante N.S.

Pendule O.R.B. n° 9

PHASES							
K1	O1	Q1	M2	S2	N2	L2	*Méth.*
—11°18	—27°58	—	—4°15	— 6°54	+ 2°53	+ 4°70	D
—13°84	—18°54	+48°62	—4°60	— 6°90	+13°38	—18°67	L
—16°30	—23°90	—26°17	—3°47	— 5°73	— 7°22	—48°27	D
—17°53	—20°39	—47°42	—3°53	— 5°03	+14°95	+ 6°02	L
—15°67	—25°00	+ 7°63	—1°50	— 3°96	+10°52	—34°56	D
—20°02	—19°63	—25°51	—3°04	— 2°56	— 3°10	—19°09	L
—17°43	—14°14	—44°40	+0°36	— 4°97	+ 9°39	+ 3°83	D
—13°05	—12°81	—45°60	—0°09	— 2°66	+ 5°58	+ 4°84	L
—11°59	—17°15	—30°22	+1°89	— 3°53	+11°62	+25°11	D
—10°34	—16°31	—42°58	+1°78	— 2°01	+ 4°05	+19°30	L
—14°00	—17°35	—34°00	+1°23	— 4°23	+ 2°25	—	D
— 9°64	—18°64	—32°11	+1°18	— 1°81	+ 0°28	+31°79	L
—13°42	—16°10	—47°71	+0°83	— 4°80	+ 2°30	—33°00	D
—14°59	—11°80	+10°61	+0°08	— 5°26	+ 4°35	—10°93	L
—23°25	—12°01	—51°90	+0°16	— 5°65	+ 2°84	— 8°06	D
—19°34	—10°78	+25°56	+0°74	— 4°06	— 6°73	+26°45	L
—28°37	—11°95	—57°74	—0°67	— 6°70	+14°48	+ 3°13	D
—30°43	—12°50	—49°98	+0°16	— 6°40	+11°69	+49°95	L
—29°85	—20°30	—25°73	—0°04	— 4°83	+ 9°02	—15°19	D
—23°64	—15°19	— 4°16	—0°02	— 5°01	+ 7°18	+19°74	L
—21°69	—17°35	—34°36	+0°36	— 6°30	— 2°33	—38°27	D
—17°83	—22°40	—22°54	—0°09	— 6°05	+ 5°50	+ 8°33	L
—11°72	—21°13	—18°00	—0°10	— 5°32	+ 7°94	—26°97	D
— 9°18	+ 8°97	+50°53	—0°64	— 5°32	+12°98	— 0°10	L

TABLE 8 (suite)

Composante N.S.

Pendule O.R.B. n° 9

Epoque			FACTEURS γ						
			K1	O1	Q1	M2	S2	N2	L2
	XII	2	2,263	2,944	0,517	0,932	1,086	0,975	1,209
		8	2,119	2,644	0,860	0,901	1,030	1,091	0,546
	XII	12	2,224	3,007	0,843	0,929	1,073	0,987	1,185
		18	2,230	3,440	3,276	0,923	1,007	0,956	0,623
	XII	22	2,309	2,881	2,421	0,897	1,024	1,014	0,234
		29	2,006	3,253	1,471	0,895	0,966	1,006	0,358
1961	I	1	2,220	3,900	2,763	0,882	1,020	0,947	0,181
		8	2,003	3,469	2,277	0,909	0,942	1,050	1,343
		11	2,215	3,934	3,270	0,901	0,978	0,848	1,855
		19	2,051	2,838	7,510	0,941	1,025	0,857	0,759
		21	2,482	3,011	6,912	0,921	1,059	0,765	1,552

TABLE 8 (suite)

Composante N.S.

Pendule O.R.B. n° 9

PHASES							
K1	O1	Q1	M2	S2	N2	L2	*Méth.*
— 7°33	—22°40	+29°60	— 0°47	— 6°22	+ 9°21	—	D
+ 5°78	— 7°80	+ 3°57	— 0°25	— 5°24	+13°64	—24°79	L
—11°05	—18°43	—51°62	— 0°33	— 5°67	+ 8°72	—	D
+ 6°47	+ 5°75	+ 0°54	— 0°31	— 6°55	+ 5°94	—	L
—21°16	—14°67	—	+ 1°63	— 7°55	— 3°36	—	D
— 8°27	+ 2°13	+ 2°73	+ 0°79	— 8°60	— 4°48	—	L
—21°43	—13°29	—21°65	+ 1°59	— 5°43	— 4°88	—	D
+ 7°40	—12°65	—40°00	+ 0°68	— 4°55	— 5°63	—	L
—15°00	—19°29	—37°37	+ 0°42	— 1°99	— 1°33	—	D
—17°99	—24°05	—11°04	+ 1°53	— 5°35	— 6°09	—	L
—25°95	—29°00	—13°96	— 0°63	— 3°97	— 5°52	—	D

In order to give some possibility of comparison, let us examine in Table 9(*) the strong variations of the ratio γ and the de-phasing α during observations in various stations.

In a general manner, we note that there is considerable difference in suitably demonstrating the diurnal waves K1 and O1 because recording from one month to the next phase differences on the order of 100° obviously must be considered a failure. However, we have been able at Sclaigneaux to limit the fluctuations of phase of the diurnal waves to less than 30° which is remarkably low when we find that this fluctuation always is on the order of 100° and often more in other stations.

Concerning the fluctuations of phase of K1 and A1 for the N-S component, it should be noted that these waves, as we have seen in chapter IV, cancel each other at latitude 45° to which most of the operating stations are relatively close. The dispersions of the waves may therefore be a function of the distance to the parallel of 45°.

Concerning the amplitude ratios, we note that Sclaigneaux seems to be the most stable in regard to the five principal waves. The Paris station also appears to be very constant. It is equipped with horizontal quartz pendulums of the Blum-Jobert type. However, we still do not possess sufficiently broad results and the N-S component is totally absent so that it is not possible to make a completely objective comparison.

The phases are generally very sensitive to errors of observation and we note here also a large homogeneity in the results of the Sclaigneaux station.

We should now examine the mean values which may be derived for the Sclaigneaux station. Of course, the waves extracted from each group of 30 days of

(*) NOTE: Table 9 missing in French text - Translator.

uninterrupted observation are not pure waves but result from the combination of a total of waves very close to each other and which cannot be separated for one month of observation. The two most characteristic cases are those of K1 and P1 and of S2 and K2.

In spite of this unfavorable circumstance, it will not be without interest to examine the mean values obtained by vectorially combining the different individual results of Tables 6, 7 and 8.

These vectorial means for each instrument and each method are grouped in Table 10.

TABLE 10

	M2		S2		N2		K1		O1		n
Component EW	$\Delta\gamma$	$\Delta\alpha$	$\Delta\gamma$	$\Delta\alpha$	$\Delta\gamma$	$\Delta\alpha$	$\Delta\gamma$	$\Delta\alpha$	$\Delta\gamma$	$\Delta\alpha$	
Kondara 1	86	9°	167	29°	439	86°	788	111°	637	125°	13
Kondara 2	221	12°	329	51°	762	125°	768	95°	1678	140°	18
Alma Ata	121	8°	208	26°	346	56°	421	75°	748	59°	13
Poltava	214	19°	379	17°	728	123°	975	132°	1087	100°	4
Ashkabad	240	36°	500	18°	830	60°	1794	112°	1004	105°	6
Berchtesgaden	118	10°	319	13°	633	102°	1281	86°	846	60°	13
Paris	58	3°	80	8°	?		567	45°	186	35°	6
Sclaigneaux (P1)	150	8°	273	14°	438	20°	374	20°	202	27°	14
Component NS											n
Kondara 1	195	39°	428	35°	1426	50°	2127	82°	1010	80°	13
Kondara 2	334	16°	509	51°	606	71°	2602	96°	4003	140°	18
Alma Ata	147	20°	720	75°	577	81°	8114	135°	4523	116°	13
Poltava	81	15°	271	26°	812	39°	2710	126°	2816	88°	4
Ashkabad	128	37°	458	62°	986	138°	4485	107°	2400	106°	6
Kounrad	43	5°	90	8°	489	37°	385	43°	605	93°	6
Berchtesgaden	165	11°	240	34°	423	73°	2215	190°	4105	166°	13
{ Sclaigneaux (P4)	129	7°	159	10°	393	16°	626	15°	738	21°	11 }
{ Sclaigneaux (P9)	50	9°	107	5°	242	21°	550	11°	387	29°	12 }
Components in other azimuths	$\Delta\alpha$ (M2)		$\Delta\alpha$ (S2)		$\Delta\alpha$ (N2)		$\Delta\alpha$ (K1)		$\Delta\alpha$ (O1)		n
Borowiec 45°	10°		73°		65°		85°		112°		9
Borowiec 135°	41°		133°		105°		154°		99°		9

(n = number of comparative analyses).

TABLE 11

				K1	O1	Q1	M2	S2	N2	L2
E.W.	P1	D	γ =	0,840	0,641	0,571	0,957	0,891	0,903	1,277
			α =	— 2°75	— 5°20	—12°97	—3°91	+1°62	—7°55	—10°93
		L	γ =	0,810	0,641	0,671	0,958	0,885	0,921	0,940
			α =	— 5°03	— 4°63	+ 9°93	—4°41	+2°58	—3°50	+ 6°98
N.S.	P4	D	γ =	2,283	2,623	4,545	0,918	1,019	0,906	1,222
			α =	—12°30	—13°07	— 8°37	+0°36	—1°27	+9°32	—12°95
		L	γ =	2,345	2,489	4,073	0,935	1,001	0,997	1,567
			α =	—12°58	—14°10	— 5°38	+1°00	—1°03	+6°73	— 1°78
	P9	D	γ =	1,880	2,709	3,315	0,940	1,072	0,973	1,047
			α =	—17°18	—17°43	—35°00	—0°21	—5°21	+5°13	—14°73
		L	γ =	1,948	2,757	2,566	0,929	1,074	1,041	0,934
			α =	—15°48	—14°15	—17°38	—0°12	—4°35	+5°78	+10°98

D: Doodson method.

L: Lecolazet method.

Chapter V

OUTLINE OF GEOPHYSICAL INTERPRETATION

It seems premature to us to draw geophysical conclusions from the result of the first year of observations at Sclaigneaux. The project under consideration consists of a prior comparison of the stations at Sclaigneaux and Warmifontaine, each of which are equipped with four pendulums.

Only two pendulums operate in each station at the present time and we have available as yet only the first results from Warmifontaine which are given further below.

The objective of this chapter will therefore be to show the way in which research should be conducted for eventually obtaining valid conclusions. Consequently any application to the Sclaigneaux data is made only as an example.

In order to attempt such a geophysical interpretation, we should initially liberate ourselves from the problem of selecting a method of harmonic analysis since small differences subsist between the results furnished by one or the other of the two analytical methods adopted (unless we have available a very long recording on the order of 2 years where these small differences then tend to disappear).

Let us provisionally admit for Sclaigneaux the general means of the two methods(*).

TABLE 13

Provisional results for the Sclaigneaux station

		K1	O1	Q1	M2	S2	N2	L2
Component EW	γ	0,825	0,641	0,621	0,957	0,888	0,912	0,940
	α	—3°89	—4°91	—1°52	—4°16	+2°10	—5°52	+6°98
Component NS	γ	2,114	2,644	3,625	0,931	1,041	0,979	1,067
	α	—14°38	—14°68	—16°53	+0°26	—2°96	+6°74	—7°09

(*) NOTE: There is no Table 12 in the French text - Translator.

The first element which strikes us is the abnormally high amplitude of the diurnal waves in the N-S component. This is not exceptional but corresponds to a general observation made throughout the entire zone of the northern hemisphere located on either side of the forty-fifth parallel. In the north-south component, these waves have a low amplitude and we might normally expect rather scattered results. However, in fact this does not involve scatter but a systematic amplification as can be seen from Table 14:

TABLE 14

Amplitude factor of diurnal waves in N-S components for various stations

	φ	K1	O1
Kondara 1	38°48'	1,555	1,523
Kondara 2	38°48'	1,462	1,918
Alma Ata 1	43°16'	4,590	2,702
Alma Ata 2	43°16'	2,039	2,835
Kounrad	46°59'	1,041	1,024
Berchtesgaden	47°38'	1,916	2,236
Poltava	49°36'	5,350	1,698
Brézové Hory	49°41'	1,424	1,579
Sclaigneaux	50°29'	2,114	2,644

In addition, the Berchtesgaden observations show a phase opposition of 180° for only the N-S diurnal waves which is even stranger. It seems scarcely logical to attribute this systematic effect to the low amplitudes of the waves which might make observation imprecise because they are of the same order of magnitude as the amplitude of wave N2 for which nothing similar is observed.

Nor does it seem that we can blame the disturbing effect of the ocean tides as an explanation for this anomaly because the diurnal waves have only a very low amplitude in the seas close to Belgium as can be seen from Table 13.

The question therefore remains unanswered.

The ORB horizontal pendulums 2 and 3 were installed by the Institute of Geophysics of Bari University in the grottoes of Castellana (Bari) whereas the ORB pendulums 5 and 6 were installed by the Institute of Geophysics of Genoa University in the city itself. Recordings are under way and numerical findings are in the process of publication by the two institutes. They should be of considerable interest especially in regard to this curious phenomenon.

We shall therefore examine in the following the significance of the results obtained for the waves M2, S2 and N2 in the two components and for the K1 and O1 waves only in the E-W component.

Role of indirect effects

In chapter III we touched on the disturbing role of an overload on the ground in a case of particular interest. A rapid flooding of the Meuse River caused an inclination of the ground under the weight of the water which was manifested by a corresponding drift of the pendulums located about 500 m from the river.

This is only a local and occasional example which seems to us, however, rather noteworthy. We shall now have to concern ourselves with the periodic disturbance induced in the movements of the soil and subsoil by the oscillatory movement of the mass of ocean waters at sometimes considerable distances from the shore. This movement is operated by the same potential as the earth tides and therefore presents the same periodicity which makes the interaction between the earth tide and these disturbing effects rather complex. The latter simultaneously present three aspects: (a) deviation from the vertical by the attraction of the mass of water; (b) inflection of the crust due to the overload (this is the most important of the indirect effects); (c) variation of the potential due to this inflection (an effect in the opposite sense to the first two which reaches about 40 % of the effect under (a) above).

These effects all have the character of a lunar-solar tide but are transmitted by the intermediary of the oceans and we can therefore say in this sense that this involves indirect lunar-solar effects whereas the earth tide is a direct lunar-solar effect.

The indirect effects are of considerable interest because their precise measurement will permit us to determine the elastic constants of large zones of the crust (because only the latter is subject to indirect effects) and to deduce from this the possible action of certain faults.

The periods of the harmonic components are consequently exactly the same in the two effects which makes the problem of their separation very complicated. Two methods can be utilized. The so-called Boussinesq method (because it is based on formulas in the theory of elasticity proposed by this author in 1878) requires detailed calculation of all the masses of water in movement (and consequently very good tidal charts) and is based on several rather summary theoretical assumptions. The method is therefore very nearly impossible to apply and raises a large number of objections by reason of the many simplifying hypotheses in it. For example, how are we to take into account the role of the underlying water tables and their reaction on the upper crust? However, such reactions have been observed, specifically by the Japanese hydrologist Kikkawa. The other and more simple method of separation is that of Corkan. It is empirical, at least in part, and based on the fact that the ratios of amplitude between the several waves are not the same in the direct effects (where they are the ratios anticipated by celestial mechanics because the earth tide is static) and in the indirect effects (because there are resonance phenomena in the oceans). This method has given coherent results and can be objected to only for a certain empiricism even though the basic hypotheses are logical and probable:

a) The several semidiurnal waves (M2, S2, N2) present in the direct effect have ratios of amplitude to each other anticipated by the static theory of the tides:

$$S_2/M_2 = 0{,}464 \qquad N_2/M_2 = 0{,}194$$

b) the several semidiurnal waves (M2, S2, N2) present in the indirect effects have the same ratios of amplitude to each other as the ocean tides which generate these indirect effects.

One of the authors has earlier discussed and analyzed the real possibilities of this method(*) and applied it to the stations at Freiberg (Saxony) and Brezove Hory (Czechoslovakia). We shall briefly resume his analysis here.

Let us assume the following:

	amplitudes	phases
Observed effect	$K = \gamma_0 E$	ζ
Direct effect	γE	0
Indirect effects	I	i

The unknown factors are γ, I, i.

For each individual wave, we have

$$K \cos(\alpha t + \zeta) = \gamma E \cos \alpha t + I \cos(\alpha t + i) \qquad (15)$$

or

$$K \cos \zeta = \gamma E + I \cos i$$
$$K \sin \zeta = I \sin i \qquad (16)$$

and consequently

$$\operatorname{tg} i = \frac{K \sin \zeta}{K \cos \zeta - \gamma E} = \frac{\gamma_0 \sin \zeta}{\gamma_0 \cos \zeta - \gamma} \qquad (17)$$

$$I = \frac{K \sin \zeta}{\sin i} \qquad (18)$$

Accordingly, to each value of γ corresponds a value of i and consequently a value of I.

(*) Paul J. Melchior: Discussion du procédé de Corkan pour la séparation des effets directs et indirects dans les marées terrestres. (Comm. Observ. Royal de Belgique no. 115, Série Géophysique no. 40, 1957).

Let us introduce, in a system of axes, I in accordance with Ox (I = x) and γ in accordance with Oy (γ = y). By eliminating the parameter i from equations (17) and (18) we obtain the relation

$$K^2 y^2 - \gamma_0^2 x^2 - 2 K^2 \gamma_0 \cos \zeta \,.\, y + K^2 \gamma_0^2 = 0 \qquad (19)$$

which represents a hyperbola ($\delta = -\gamma_0^2 K^2$) whose center is situated on Oy and has as ordinate $\gamma_0 \cos \zeta$ and whose summit has as coordinates ($K \sin \zeta$, $\gamma_0 \cos \zeta$). Its axis is parallel to Oy($x = \gamma_0 \cos \zeta$). We need here consider only the branch located in the field I > 0.

Having thus extracted, by harmonic analysis of the observed curves, the constants (K, ζ) for each of the waves M2, S2, N2, we are able to construct a hyperbola (19) which is characteristic of each of these waves. However, these hyperbolas cannot be compared among each other because the scale of x differs in each case whereas the scale of the y is common to them. (We have taken into account the coefficient γ and not the amplitude of the wave which implies the hypothesis a).

In order to allow mutual comparison, we need to introduce the hypothesis b) and multiply the amplitude (I = x) of the wave S_2 of the indirect effects by the coefficient of reduction of this wave in the representative ocean (or about 3) and the amplitude of the wave N_2 of the indirect effects by the coefficient of reduction of this wave in the respective ocean (or about 5.2).

We thus obtain two independent determinations of the indirect effect causes by the same mass of water (but acting with different periods and phases). The coordinates of the point of encounter of the two branches of the hyperbolas characteristic of M2 and S2 are the value of I (M_2) and that of γ.

The same is true for the points of encounter of the hyperbolas (S_2) and (N_2) and of the hyperbolas (M_2) and N_2).

Actually, two of the branches of the characteristic hyperbolas may encounter each other at two points and we will then have two solutions of which one is rejected if it gives an abnormal value for γ (outside of the field (o - 1)). We can also eliminate one of the solutions if it produces values of the phase i of the several waves in flagrant disagreement with the phases of the ocean tides which generate these indirect effects. This is easier for a coastal than for a continental station.

It may also happen that two hyperbolas do not have any point of encounter in the field $o < \gamma < 1$ (this is the case for the waves S_2, M_2 at Freiberg and at Brezove Hory in the north-south direction).

The solutions of the system formed by the three equations of type (17) corresponding to the three waves M_2, S_2, N_2 obviously depend on the numerical values adopted for the ratios S_2/M_2 and N_2/M_2.

These are difficult to establish because they imply that we know a priori the exact role held by each ocean region in the indirect effects in order to be able to correctly weight the observed values for these ratios in these oceans and thus obtain a significant mean value.

After observing the appreciable differences between the North Atlantic, the North Sea, the Irish Sea and, on the other hand, the Adriatic, the Mediterranean, the Baltic Sea and the Arctic Ocean, Melchior suggested adopting the following coefficients for the stations of Central Europe:

In the meridian	M2/S2 = 2.5	M2/N2 = 4.9
In the first vertical	M2/S2 = 3.5	M2/N2 = 5.2

The very coherent results obtained for the Brezove Hory and Freiberg stations through these coefficients seem to validate their selection.

However, it is obvious that their value may depend on the geographic location of the station in relation to the distribution of the seas and oceans.

Moreover, we may consider that the determination of the optimum coefficients should result from the observations themselves (conjugated direct and indirect effects) and this would be possible and extremely interesting if we had available a satisfactory network of stations well distributed over Europe.

Provisionally, we shall utilize the same coefficients for Sclaigneaux.

Semidiurnal waves

North-south component.

The de-phasing of M2 is so minor that we can state $\zeta = 0$, $i = 0$ which reduces the characteristic hyperbola to a straight line.

This straight line encounters the hyperbola of N2 at the point

$$\gamma = 0,77 \qquad I\,(M2) = 1,617 \text{ mseca}$$

for which $i_{N2} = 1.6^{o}$ which is sufficiently coherent since the phase of the ocean tide N2 is zero between Ostende and Flessingue.

The hyperbola of S2 does not intersect those of M2 and N2 which may indicate that the coefficient M2/S2 utilized is not accurate for the Sclaigneaux station as was the case for Freiberg and Brézové Hory. The findings at Warmifontaine may doubtlessly help in resolving this question.

East-west component.

The hyperbola of M2 encounters the hyperbola of S2 at the point

$$\gamma = 0,76 \qquad I\,(M2) = 2,19 \text{ mseca}$$

for which

$$i_{M2} = 19,5^{\circ} \qquad i_{S2} = 13,9^{\circ}$$

which is still very coherent because the phase of the ocean tide M2 is 20° between Ostende and Flessingue and that of S2 is 15° between Le Havre and Dunkerque. In this case, the hyperbola of N2 does not intersect the others which indicates that we need to revise the coefficient M2/N2 for the east-west component.

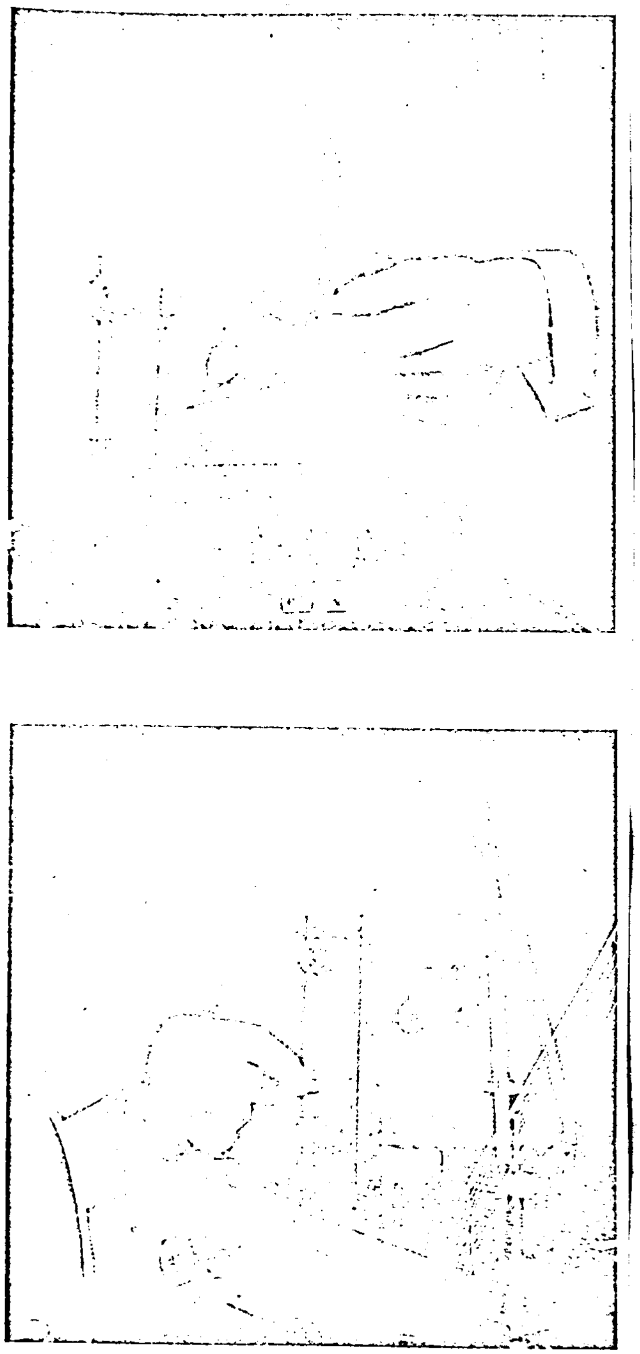


Fig. 55 - ORB pendulum 15.

This type-II pendulum is equipped with the new locking cradle for absolute stability during transport. Release of the pendulum arm is both simpler and less delicate because it is made in only two stages shown on the two sketches above. The operator initially rotates the rear pivot of the lock (top sketch) and subsequently lowers the entire locking system by lightly unscrewing the forward wedge button (bottom sketch).

Diurnal waves

The diurnal ocean tides are so minor in the European oceans (as shown in Table 15) that we may consider the factors of amplitude of K1 and O1 for the east-west component as free of indirect effects on the average. The mean of K1 and O1 is

$$\gamma = 0.733$$

THE FACTOR γ

We therefore note a definite coherence between the preliminary results obtained which furnish a value in the neighborhood of 0.75 for the factor γ at Sclaigneaux.

Let us remember that, in the publication already cited, the following values have been obtained by Melchior:

Freiberg	$\gamma = 0.704$
Březové Hory	$\gamma = 0.709$
Bidston	$\gamma = 0.703$

We may also consider that the stations situated in the center of a large continent are very nearly free of these indirect effects.

There presently exist two stations in Russia for which not less than one year of observations has been analyzed and published(*). The mean of the values obtained for the factor γ by the wave M_2 appears as follows

		NS	EW	
Alma Ata	1	0.657	0.704	0,705
	2	0.724	0.734	
Kondara	1	0.920	0.565	0,722
	2	0.771	0.645	
	3	0.801	0.633	

We may therefore consider that the true value of the factor γ is slightly higher than 0.7.

(*) N. N. Pariiskii. - Observation of the Earth tides in the USSR from June 1957 to June 1960. (Bull. Inf. Marées Terrestres, no. 21, pp. 371-386, Uccle 1960).

First results of the Warmifontaine station

A first harmonic analysis was performed for the first month of operation of our new station.

Composante E.W. *Pendule O.R.B. n° 11* Période 73 s 17
Epoque 1961 février 24, 4 h TU (L), 7 h TU (D) s = 0''001094
σ = 0,273 mm = 0''000298

	K1	O1	Q1	M2	S2	N2	L2	
γ	0,754	0,700	0,736	0,742	0,637	0,808	0,574	L
α	10°54	15°87	—12°68	—3°01	16°72	3°24	44°56	
γ	0,737	0,671	0,630	0,727	0,644	0,656	0,886	D
α	10°26	15°30	—33°44	9°18	18°92	4°08	44°39	

Legend:

"Composante" - Component
"Pendule" - Pendulum
"Période" - Period
"Epoque" - Time
"février" - February
"T U" - Universal time

AMPLITUDE OF INDIRECT EFFECTS

It is of interest to see that the component M_2 of the ocean tides in the European oceans whose amplitude is 1 or 2 m, induces inflections of the continental platform whose slope at Sclaigneaux is 1.6 mseca in the north-south direction and 2.2 mseca in the east-west direction.

For the central European stations, P. Melchior found at Freiberg 1.3 mseca in both directions and at Brézové Hory 1.2 mseca in the east-west direction and 0.7 mseca in the north-south direction. We thus see that the indirect effects diminish at the rate of distance from the sea.

The new Central European stations (Berchtesgaden, Tiefenort and Berggiesshübel) will soon add to our knowledge.

However, it would be most desirable to complete the network by installing two new stations to close the gap between Belgium and Central Europe.

TABLE 15

	M2		S2		N2		K1		O1		M2/S2	M2/N2
	H	g	H	g	H	g	H	g	H	g		
Brest	204.48	100°7	74.79	139°1	43.01	77°2	6.43	71°9	6.69	323°0	2.734	4.754
Cherbourg	186.98	227°0	69.38	269°9	36.49	213°7	9.33	107°2	6.93	356°8	2.695	5.124
Le Havre	263.15	285°1	87.03	332°0	52.34	265°7	9.87	117°4	5.15	8°6	3.024	5.028
Dover	224.44	330°	70.92	21°	42.72	310°	4.24	39°	7.04	175°	3.165	5.254
Dunkerque	208.02	357°8	61.61	49°6	37.62	332°0	3.91	355°5	6.58	170°3	3.376	5.529
Ostende	175.68	5°0	51.99	58°3	28.84	338°8	4.30	346°4	8.08	173°7	3.379	6.092
Flessingue	167.94	42°	45.83	97°	27.78	24°	6.77	6°	10.61	191°	3.664	6.045
Hansweert	186.75	68°	47.90	128°	30.76	43°	7.29	24°	11.68	199°	3.899	6.071
IJmuiden	65.13	116°	16.28	181°	9.15	102°	7.71	355°	11.43	187°	4.001	7.118

The H-amplitudes are given in cm.

Chapter VI

NEW RESEARCH UNDER WAY

1. Automatic calibrating device for horizontal pendulums

Calibration as described in chapter II is rather time consuming and requires each time the full attention of an operator for about 1 hour. The main complication which appreciably prolongs the duration of the operation is due to the fact that it is impossible to manipulate manually with sufficient slowness and uniformity the mercury bucket of the flexible tubing so as not to induce through abrupt movement the appearance of stray oscillations in the movement of the pendulum arm. After each manipulation of the mercury bucket, it is therefore necessary to attenuate these oscillations before being able to accurately read the position of the image of the luminous trace on the graduated horizontal scale. In order to avoid the loss of time inherent in this procedure for calibration and to also increase the accuracy of the determination by eliminating the presence of an observer in the vicinity of the pendulum, calibration has been made entirely automatic by utilizing the method of the turntable already suggested in chapter II.

Oscillatory micromovements of the base of the pendulum of a strictly given amplitude and arbitrarily selected period (for example, one hour) are induced by means of the expandable crapaudine and the rotating device. These movements follow a perfectly sinusoidal law as a function of time and generate the amplified movements of the pendulum arm which are ultimately recorded by the standard photographic method on a drum placed at a distance of 5 meters and whose period of rotation is here reduced to 6 hours. The changes of level of the mercury bucket are thus generated mechanically with great uniformity and no longer introduce appreciable disturbance; since their values are a sinusoidal function of time, the curves recorded are generally very clear sinusoids of a quasi-constant amplitude.

However, if the ground supporting the column on which the pendulum rests is agitated by movements resembling micro-earthquakes which is usual during the day in the vicinity of a large city, we note on the recording a composite curve which results from the superposition of the two oscillatory movements. The first has great amplitude and a period of one hour and obviously corresponds to the oscillations imposed on the pendulum. The second has low amplitude and becomes even unnoticeable at certain moments when the ground is particularly quiet, which is sometimes the case in the middle of the night, and corresponds to the vibrations of the pendulum itself excited by the trembling of the ground. The curve recorded then presents itself as a slightly looped sinusoid. Since the presence of these loops manifest the inherent oscillations of the pendulum, it is of considerable advantage because it allows us to determine accurately the mean duration of a free oscillation of the pendulum and this a datum which is necessary for determining the coefficient of sensitivity of the instrument. It is sufficient to have available on the recording a scale of time, to count between two given hourly marks the number of loops in order to calculate a mean value which will be very accurate if the interval of time is sufficiently long.

The possibility we have of being able to count a rather large quantity of free oscillations of the pendulum (in general, on the order of 200) furnishes the method of photographic calibration with a clear superiority over the manual method where we can count not much more than 20 oscillations in order to define the period.

The many calibrations already made with this method have always been carried out in the laboratory and consequently under conditions of thermal and ground stability less favorable than in an underground installation. Accordingly, the measurements are always slightly disturbed by a more or less fluctuating drift. However, the mean of any number of values which can be obtained automatically without any loss of time allows us to arrive at a very accurate determination of the coefficient of sensitivity. The observations can be made without difficulty during the quiet hours of the night between midnight and 6:00 in the morning since they do not require the presence of an operator. A timing switch makes it possible to start and stop at the desired time all recordings and the operation of the crapaudine (Fig. 56).

The following is a brief description of the rotary device for imparting to the mercury bucket a uniform circular movement by simultaneously providing the time marks which make it possible to localize the position in height of the bucket in relation to the photographic recording. The device (Fig. 56) was designed and built by Mr. Geerlandt, chief technician at the Observatory.

The rotary system itself consists of a horizontal shaft resting in two ball bearings spaced fairly widely apart to reduce any possible vibration. The shaft is driven by a small synchronous motor mounted directly on one end as is the case also for the recorder (cf. Fig. 44).

The motor is geared to effect one complete revolution in 60 minutes. On the other end of the shaft, a ratchet wheel is centered on the latter and carries an even number of pins activating an electric switch. Two of these pins exactly opposite to each other have been removed in order to indicate completion of one full revolution. The contact device is intended to black out (except during the passage of the removed pins) the luminous spot so as to indicate a scale of time by brief interruptions in the recorded curve. On the same side, a rather large metallic straight edge is attached in its center to the end of the shaft and perpendicular to the latter. The straight edge is provided on either side of the center with threaded holes spaced every 5 cm in which two steel pins can be inserted symmetrically which always remain

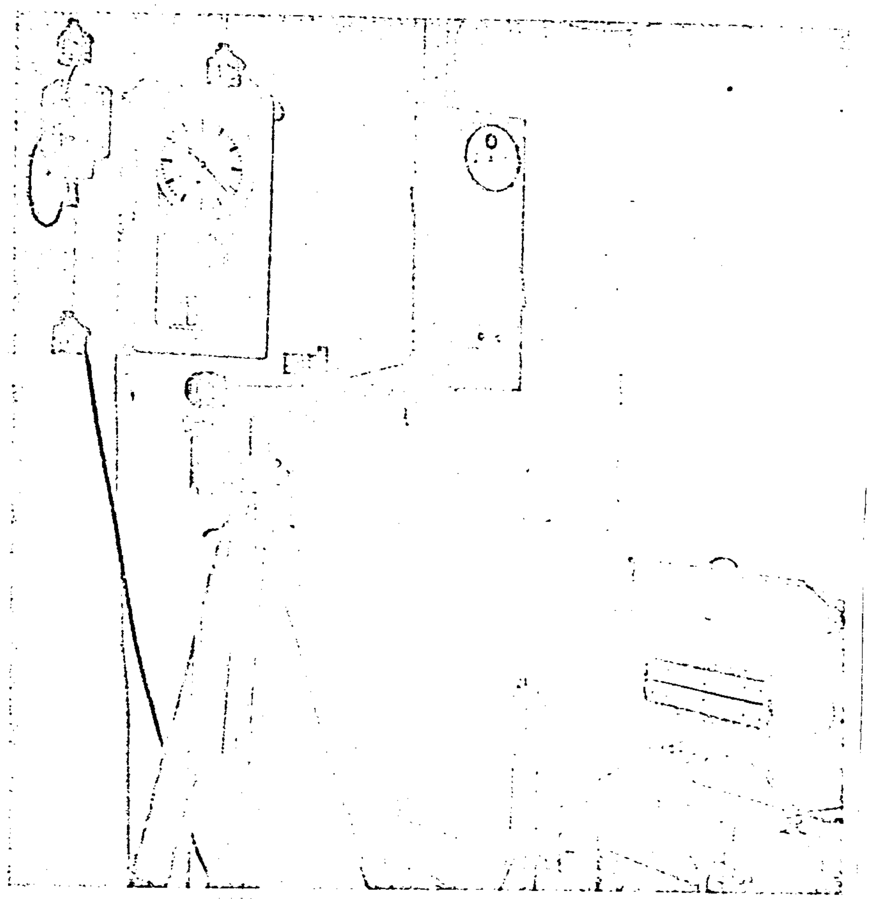


Fig. 56 - Automatic calibration equipment installed at one of the stations of the Royal Belgian Observatory. Note the movable arm device carrying the mercury bucket mounted on a tripod. It is activated by the small motor at the end of the shaft. A preset clock transmits luminous signals each quarter hour. At the bottom on the right, a recorder and projection system are placed at a distance of 5 m from the pendulum. The preset timing switch at the left of the clock energizes the entire assembly and interrupts operation at preset times.

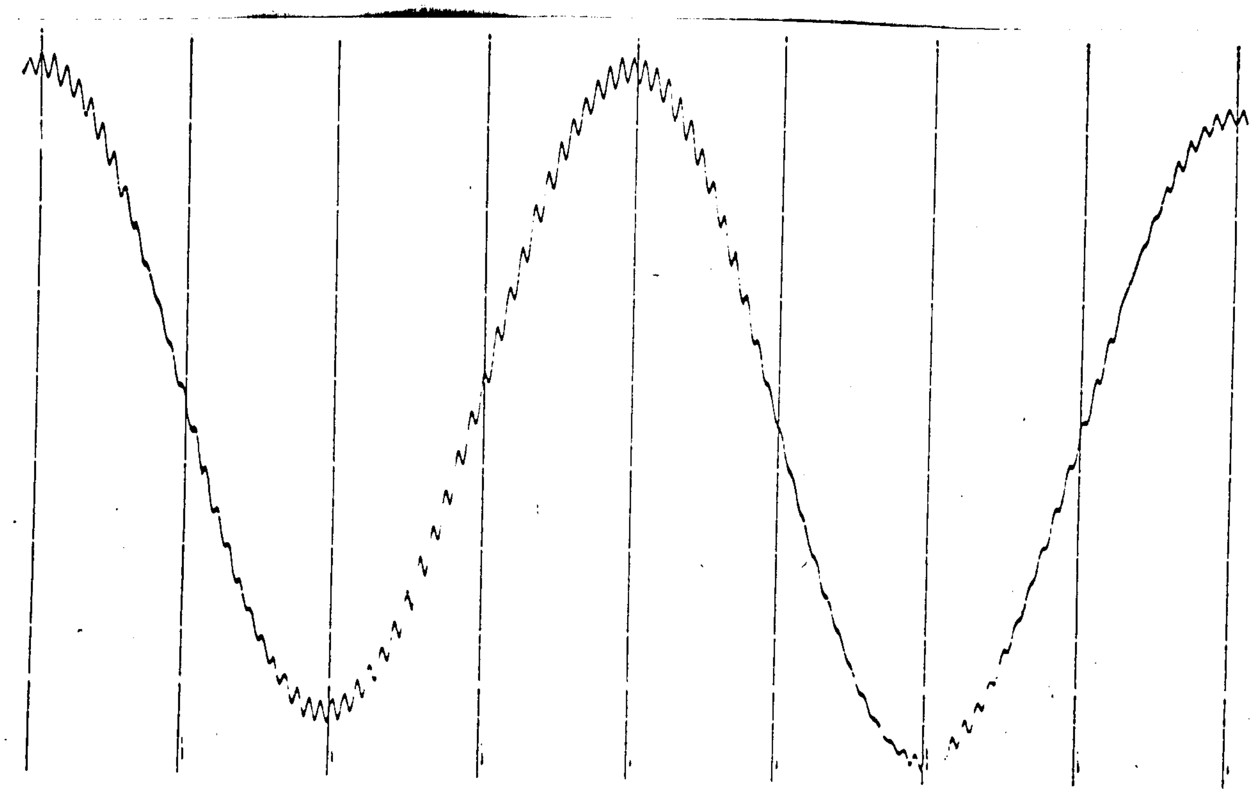
horizontal. One of these serves for suspending from a ring the mercury bucket which is thus carried along in a circular movement of a more or less large radius and the other serves for suspending a counterweight which balances the bucket. In spite of the low power (2 W) of the motor, the bucket moves with perfect uniformity and above all without any vibration.

On the edge of the straight edge is a small bubble level intended to accurately localize the instant where the straight edge is in a horizontal position during rotation. This is also the instant where the speed in a vertical direction of the mercury bucket is greatest and the moment where, if there is no de-phasing, the pendulum arm should have zero elongation. It is very important to be able to localize on the recording the point which corresponds to this instant. The ratchet wheel is held by slight friction in its support and its position can be regulated so that the first pin following the removed pins activate precisely at this instant the characteristic contacts. These adjustments can be made easily and accurately with the aid of the bubble level.

The theoretical position of the summits of the sinusoid on the graphic can be accurately found by counting on the curve a number of breaks in continuity starting with those relative to the two characteristic contacts and equivalent to one-fourth of the total number of pins.

During recent experiments, we found it sufficient to use a wheel activating only 4 contacts spaced every 90° so as to accurately localize only the position of the summits of the sinusoids as well as the points de-phased by $\pm 90^{\circ}$ in relation to the summit.

In Fig. 56, the system is mounted on a tripod which permits rapid installation in an underground station if we desire to effect on location a check of the sensitivity of a pendulum that has been in operation for a given time. In the laboratory, it is preferable to use a stable wall mounting better adapted to systematic work for any number of purposes. Independently of the calibration, the same system makes it possible to experimentally investigate quite a number of problems concerning the operation of the horizontal pendulums, for example, investigation of the variation of sensitivity as a function of the elongation often pointed out by other authors, the study of the methods of attenuation, and the demonstration of the respective qualities of various suspension wires, etc.



<u>Fig. 57</u> - Section of automatic calibration recording.
The vertical lines mark the quarter hours; the short period oscillations make possible measurement of the characteristic period of the pendulum whereas the large sinusoid represents the oscillation induced in the pendulum by the expandable crapaudine (scale of sketch: $\frac{1}{2}$).

As was to be expected, the table of calibration of pendulum 12 showed that the scatter of the automatic measurements is less than that of the manual measurement. At the same time as greater ease and an enormous gain in time, the automatic device consequently provides greater precision. However, this precision is still limited by the drift of the pendulum which fluctuates, over an interval of six hours, particularly by reason of variations of temperature. It would be necessary to install the device in an underground station to determine the actual threshold of precision which it is possible to attain by this means.

2. Calibration of crapaudines by interferometry

The value of the coefficients of calibration of the crapaudines obtained by interferometry obviously also plays a primary role in the determination of the final values for the pendulums themselves.

If the initial precision in such interferometric measurements is insufficient, this results evidently in an error of precision in all of the pendulum coefficients obtained by the method.

Up to the present, the interferometric determinations have been based exclusively on the green line of mercury $\lambda = 0.546\ \mu$ obtained by filtering the mercury light by means of an interferometric filter. We are now arranging ourselves, in order to increase the precision of these fundamental determinations and to have a control of the measurements, to also use the red line of cadmium already frequently employed in interferometry and eventually still other lines.

Recently, the interferometer itself has been equipped with plane-parallel semi-silvered glass plates replacing the transparent plates used previously.

We thus have available multiple reflections for reducing interference and the fringes have a remarkable fineness which tends to increase the precision of these fundamental determinations.

ORB Pendulum no. 12 - Calibration table

A) Calibration with "manual" crapaudine

Period	K
64,27	6,4172
64,06	5,9959
67,02	6,3647
67,23	6,2600
67,93	6,0857
	6,22470

B) Calibration with "automatic" crapaudine

Period	K
73,684	6,1988
73,708	6,3283
73,770	6,2272
74,168	6,1446
74,292	6,2394
74,522	6,3518
77,343	6,1143
79,882	6,1409
80,769	6,3740
	6,23548

3. Inflection of ground under the influence of surface loads

An investigation of the drift of the Sclaigneaux pendulums related to the variations of water level of the Meuse River is under way.

Mr. G. Henrard, chief engineer of Ponts et Chaussées at Namur furnished us with readings of the water level at the Meuse River locks and specifically at the Sclayn gate which is very close to the entrace of the gallery Sainte-Barbe at Sclaigneaux.

In order to demonstrate the actual movement of the ground in the drift curves of the pendulums, we have initially approximated these curves by a polynomial on the order of 4 and, for the period from September 1960 to March 1961, calculated these polynomials on the basis of about 200 points at the ratio of one point per day. The calculation was obviously carried out with the IBM-650. We have already referred to the expression of these polynomials in chapter III while discussing the stability in time of the instruments. It does seem probable that this uniform and systematic drift may be of instrumental origin and, as long as we do not have two instruments at each station, it would be risky to give this any geophysical significance.

At the present time, our attention is occupied by the divergence of the actual drift in relation to this parabola of the fourth order. These divergences appear closely linked to the fluctuations of level of the Meuse River.

During the winter 1960-61, the river carried high water of varying volume on 5 November and 6 December 1960, and on 11 January and 1 February 1961. Minima were recorded on 25 November and 26 December as well as on 27 January 1961.

We find that the curve of the divergence of drift of the two pendulums is practically parallel to that of the river level at Sclayn. The detailed curves just plotted are still being examined because it appears that there is a certain lag in the reaction of the pendulums.

This detailed study of the drift and its relation with the level of the Meuse River could be carried out with the assistance of the Belgian National Scientific Research Fund which made the IBM-650 available during the entire time required for calculation.

In conclusion, grateful acknowledgment is due to our many collaborators. The success of the experiments described is in large part due to their devoted and enthusiastic aid.

Mr. A. Vandewinkel, mathematician at the International Center of Earth Tides, has contributed to all the phases of installation and operational readying of the stations at Sclaigneaux and Warmifontaine. Together with Mr. J. Verlaine, mathematician at the same center, he carried out the analysis of the recordings and the preparation of these data for calculation.

The exploitation of the results furnished by the computer was carried out by Messrs. H. Bernard and M. Vandenbauw, mathematicians at the Royal Belgian Observatory, and by Messrs. Vandewinkel and Verlaine. All of them also carried out many calibration operations.

In regard to the technical aspects, we have already mentioned that the horizontal pendulums were constructed by Mr. F. Bourdon, Chief Designer, while the construction of the recorders, projectors and of the automatic calibration device is due to Mr. J. Geerlandt, Chief Technician.

The electrical wiring and installation work was carried out by Mr. Y. Decoeur, Chief Designer, and Mr. Demily, Chief Technician. Finally the greater part of the photographs as well as a documentary film presented to the General Assembly of UGGI at Helsinki, as well as to the Belgian Society for Astronomy were made by Mr. F. Van Cuyck, mathematician at the Royal Belgian Observatory.

Brussels, 15 May 1961